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Jeffery Slip Fluid Flow with the Magnetic Dipole **Effect Over a Melting or Permeable Linearly Stretching Sheet**

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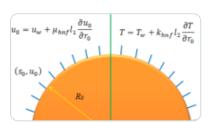
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Abstract

The magnetic dipole effect has garnered significant attention in recent years due to its potential impact on fluid flow phenomena. Researchers have explored its applications in areas such as microfluidics, nano-fluidics, additive manufacturing, drug delivery systems, magnetic resonance imaging, and hyperthermia treatments. The objectives of this research are twofold: first, to investigate the fundamental mechanisms and behaviors of Jeffery fluid flow under the influence of a magnetic dipole and slip effect; and second, to

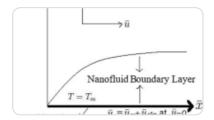
explore heat transport under the influence of melting effect and explore implications of this effect in various fields. Here we consider two different types of surface of fluid flow namely melting surface and permeable surface. By using the well-known similarity transformation, formulated flow equations are converted into OD equations and solved numerically using R-K 4th order with shooting techniques in Matlab. Graphical representations are used to show how different physical characteristics affect velocity and temperature profiles. The findings demonstrate that the velocity profile increases over a range of Deborah numbers (\(\gamma_{1}\)), whereas the temperature profile exhibits the opposite behavior. Velocity profile gets cut down for diverse values of ferromagnetic interaction parameter (\(\beta\)) but on the other hand temperature profile accelerates. The investigation also revealed that the viscous dissipation parameter λ had counterintuitive effects on the thermal profile. Deborah numbers \(\gamma_{1}\) on velocity $(f^{\rho } \left(\frac{\pi \left(\pi \left(\frac{\pi \left(\frac{\pi \left(\pi\right) }{\pi \left(\frac{\pi \left(\frac{\pi \left(\frac{\pi \left(\pi\right) }{\pi \left(\frac{\pi \left(\frac{\pi \left(\pi\right) }{\pi \left(\frac{\pi \left(\frac{\pi \left(\pi\right) }{\pi \left(\pi\right) }{\pi \left(\pi\right) }{\pi \left(\pi\right) }{\pi \left(\frac{\pi \left(\pi\right) }{\pi \left($ profile. Whenever values \(\gamma_{1}\) get increased the velocity gets enhanced but alternatively temperature profile gets cut down. In this article, we find the tabulated form the numerical values of skin friction coefficient, Nusselt number in PST is given for numerical solution on melting surface case and without melting case for various values of the physical parameter. The graphically results show that the melting surface influence more than permeable surface.

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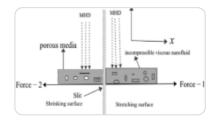
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Introduction

Non-Newtonian fluid flow stands up in many divisions of chemical and material processing engineering. There are various types of non-Newton fluids like Viscoelastic fluid, couple stress fluid, micropolar fluid power-law fluid, etc. In addition to these, there is another non-Newtonian model called the Jeffery fluid model. The stress relaxation feature of non-Newtonian fluids, which the typical viscous fluid model cannot represent, may be described using the Jeffrey fluid model. The framework of Jeffrey fluid may adequately explain a type of non-Newtonian fluids with different temporal scales for memories, also known as the relaxing period. The Couette and Poiseuille flows of a Jeffrey fluid under slip boundary conditions were the subject of an investigation by Ramesh [1] into the effects of viscous dissipation and Joule heating. Abbasi et al. [2] explained the interaction between Jeffrey nanofluid's mixed convection flow, thermal radiation, and double stratification. Shehzad et al. [3] investigated Jeffrey nanofluid thermally radiative three-dimensional flow with internal heat generation and field of magneticAccurate analytical solutions for the transport of heat and flow on a stretching/shrinking sheet close to the stagnation point in a Jeffrey fluid were provided by Turkyilmazoglu et al. [4]. Ellahi et al. [5, 6] examined the effect Bloodstream of Jeffrey liquid in a catheterized tightened supply route with the nanomaterials suspended and also investigated the Peristaltic transport of Jeffrey fluid in a rectangular tube. Hayat et al. [7] investigated the erratic flow as well as heat transfer of the Jeffrey fluid around a stretched sheet. Ahmed et al. [8] studied the convective heat transfer of the MHD Jeffrey fluid around a stretched sheet.

A model of electrically conducting fluids known as magnetohydrodynamics (MHD), sometimes known as magneto-fluid dynamics or hydromagnetics, considers all interpenetrating particle species as a single continuous medium. It has supplications in a deep range of regulations, involving geophysics, astronomy, and engineering, and is principally focused on the lower-frequency, wide range on the scale, magnetic behavior in plasmas and liquid metals. Ogulu et al. [9] modeling of pulsatile blood flow in a homogenous porous bed with a consistent magnetic field with time-varying suction was conducted. Alam et al. [10] look into the blood flow and the transfer of heat using gold nanoparticles around a stretched sheet when a magnetic dipole is present. In the situation of an unstable flow, Seddeek [11] investigated the outcomes of radiation together with changing viscosity on an MHD-free convection flow across a leveled plate with a semi-infinite length and an aligned magnetic field. Precise logical solutions for the heat and

mass transfer of MHD slip flow in nanofluids were discovered by Turkyilmazoglu [12]. The outcomes of heat radiation, suction/blowing, and slide-on boundary layer flow of magnetohydrodynamics across an exponentially stretched sheet were looked over by Mukhopadhyay [13]. Raptis et al. [14] describe the outcomes of heat radiation on MHD flow. Iyoko et al. [15] Investigation of magnetic dipole and thermal radiation effects on Jeffery flow/heat transfer across a strain plate by suction/injection.

The procedure of melting is commonly utilized in machinery such as metal casting, laser manufacturing, wandering freezing, soil melting, rivers and lakes, etc. Singh et al. [16] looked into the effects of melting heat transfer in boundary layer stagnation point flow of MHD micro-polar fluid towards a stretching/shrinking surface. By employing carbon nanotubes, Hayat et al. [17] inspected the numerical analysis for melting heat transmission and homogeneous heterogeneous reactions in flow. Melting heat transfer in continuous laminar flow across a flat plate was given by Epstein et al. [18]. Melting heat transfer in constant laminar flow around a moving surface was given by Ishak et al. [19]. In a micropolar fluid, Yacob et al. [20] investigated the impression of melting heat transfer in boundary layer stagnation-point flow in the direction of a stretching/shrinking sheet. Olkha and Dadheech [21, 22] discussed entropy analysis for MHD flow for different non-Newtonian fluids caused by a stretching sheet with melting and slip effects. Dadheech et al. [23] investigated MHD flow for Casson fluid caused by a stretching sheet with melting and slip effects. Dadheech et al. [24] discussed entropy analysis for Williamson fluid caused by a vertical plate with Cattaneo-Christov heat flux and slip effect.

Although a fluid typically sticks to solid boundaries (has no slip), there are several circumstances in which this is not the case. For instance, suspensions, polymer melts, emulsions, and many other non-Newtonian fluids frequently show macroscopic wall slips. These fluids with boundary slip have uses in cleaning interior cavities, prosthetic heart valves, and several other technical operations. The slip effect on non-Newtonian fluid flows was investigated by Labropulu et al. [25]. Ali et al. [26] inspected slip effects in viscoelastic fluid flow caused by an oscillatory stretched sheet through a porous medium. Govindarajan et al. [27] discussed slip as well as mass transport effects in a vertical channel under consideration of heat source and radiation. The slip flow of Maxwell fluid past a non-linearly stretchy surface was examined by Dawar et al. [28]. Similar work has been studied by Dadheech et al. [29], Olkha et al. [30].

In light of the provided literature research, we have noted that there are just a few investigations on Jeffery fluid flow with magnetic dipole effect. The main objective of the current study is to determine flow behavior and heat transfer of Jeffery fluid flow with magnetic dipole effect. The novelty of the presented work is increased by substantial validating slip effects with radiation and heat source effects. The examinations furnished in the given article can be further utilized to make investigations in microfluidics, biomedical engineering, industrial processes (e.g., polymer manufacturing), and geophysics/astrophysics. It helps in the manipulation of micro-objects, drug delivery systems, process optimization, and understanding of magnetized fluids in Earth's core and stellar interiors. The work provided in the study has not yet been disseminated, to the best of the author's knowledge.

Mathematical Formulation

Jeffery flow with the magnetic dipole field, magnetic scalar potential is taken as

Here \(\gamma\) is the magnetic field strength, the component of x and y axis direction of the magnetic field are $\(H_{x}\)$ and $\(H_{y}\)$.

```
\ H_{x} = -\frac{\hat x} = -\frac{\pi x} = \frac{\pi x}
```

```
 $$ H_{y} = -\frac{\left( \right)}{{\left( x^{2} + (y + a)^{2} )^{2} }} \right) = \frac{2\pi }{2\pi }\left( {\frac{2x(y + a)}{{(x^{2} + (y + a)^{2} )^{2} }} \right)}, $$ (3)
```

The resultant magnitude H of the magnetic field intensity is

(4)

```
\  \frac{\hat X} = - \frac{2\pi }{(y + a)^{4} } \right],
```

(5)

```
$$ \frac{\partial H}{{\partial y}} = - \frac{\gamma }{2\pi }\left( {\frac{ - 2}{{(y + a)^{3} }} + \frac{{4x^{2} }}{{(y + a)^{5} }} \right)\,, $$

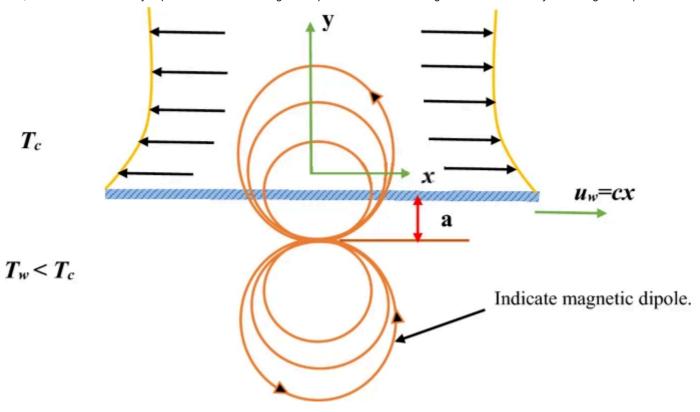
(6)
```

Magnetization M could be a "linear function of temperature",

```
$ M = K^{*} \left[ {T_{c} - T} \right]\,\,, $$
```

Let us assume the steady 2-D flow passing through a stretchy surface shown schematically in Fig. 1. By applying two equal and opposite forces along the x-axis, the sheet being stretched is related to how far it is from the fixed origin at x=0. Thus, only the moving sheet is responsible for the resultant motion of the otherwise quiescent fluid. A magnetic dipole is found some distance below the surface, while an incompressible, viscous, and electrically non-conducting ferrofluid is limited to the half-space y>0 over the sheet. The dipole, whose center lies on the y-axis a distance below the x-axis and whose magnetic field points in the positive x-direction, rises to a magnetic area of sufficient strength to saturate the ferrofluid. The stretching sheet is kept at a constant temperature (T_{w}) lower than the Curie temperature (T_{c}) , while the fluid components are far off from the surface which is presumed to be at temperature (T_{v}) and, hence, it is powerless to be magnetized until they start to cool upon entering the thermal boundary layer closest to the surface.

Fig. 1



The geometry of the problem

The required equations for the Jeffrey model are written as

$$$$$
 \tau = pI + \kappa \$\$

(7)

 $\ \ = \frac{1} }\left[{R_{1} + \lambda_{1} }\left[{R_{1} + \beta_{1} }\left[{R_{1} + \beta_{1} }\right] \right] \right]$

(8)

$$\ R_{1} = (\nabla V) + (\nabla V)^{\prime} \$$

(9)

"Cauchy stress tensor:\(\tau\),extra stress tensor:\(\kappa\), and the following terms are defined above paragraph.

Using the continuity, momentum, temperature equations, and the boundary conditions"

```
\ \frac{\partial u}{{\partial x}} + \frac{\partial u}{{\partial y}} = 0 $$
```

(10)

(11)

where $\langle (u(x,y) \rangle)$ and $\langle (v(x,y) \rangle)$ are the "horizontal and vertical velocity components."

Following Rosselandestimate \(q_{r}\), the radiation heat flux is given \(q_{r} = - \left(\frac{4 \right)^{4}} \right)^{1} \left(\frac{4^{r}} = - \left(\frac{4^{r}} - \left(\frac{4^{r}} \right)^{4}} \right)^{4} in a Taylor series about \(T_{\infty}\), on neglecting higher order term and \(k^*\): thermal radiation parameter, we get

 $\label{thm:linear} $$\left[T^{4} & \operatorname{T_{\leftinfty}^{4} + 4T_{\leftinfty}^{3} T - 4T_{\leftinfty}^{3} T_{\leftinfty} \right] \right] $$ $$\left[T_{\leftinfty} \right] $$ = \frac{\pi }{\left[T^{4} \right] } \right] $$ = \frac{\pi }{\left[T^{4} \right] } $$ = \frac{\pi }{\left[T$

 $\label{thm:conditional} $$ \left(- 16 \right) ^{3} }_{{3k^{*}}} \frac{(\partial^{2} T)}_{{\partial y^{2}}} \leq (\partial^{2} T)}_{{\partial y^{2}}} \right) $$$

Following boundary conditions for PST and PHF cases are given below:

```
$\$ \left[ u = (u_{w} = cx) + L_{1} \left( u_{y}, \quad v = -v_{w} + \frac{k}{{\rho \left( x_{s} (T_{m} - T_{0}) \right)} \right]} \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right] \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0}) \right) \right\} \\  \{\{\rho u = (u_{w} = cx) + L_{1} \left( x_{s} (T_{m} - T_{0
```

```
\ensuremath{\$} \Rightarrow $\left\{ 20}l \right\} T = T_{w} = T_{c} - A\left( \frac{x}{l} \right)^{2} \right. \\ \left\{ at\; y = 0 \right\} \left\{ T \setminus T_{c} \right\} \left\{ at\; y = \inf y \right\} \left\{ x\right\}
```

Solution

The similarity conversions and dimensional variables which is utilized to change Eq. $(\underline{11})$ – $(\underline{13})$ in the set of ODE are considered by Majeed et al. $[\underline{6}]$ and Wang et al. $[\underline{31},32,33,34,35,36]$.

```
\label{thm:condition} $$ \left[ \frac{c}{nu} \right] y,\quad x = \left[ \frac{c}{nu} \right] x,\quad u = \exp^{\left(\frac{c}{nu}\right)} x,\quad u
```

where in perspective of Eq. ($\underline{14}$), the Eqs. ($\underline{11}$)–($\underline{13}$) and the boundary conditions are converted to

```
\f^{\operatorname{prime prime prime } - (1 + \lambda_{2})(f^{\operatorname{prime 2} - ff^{\operatorname{prime 2} - ff^{\operatorname{prime 2} - ff^{\operatorname{prime 2} - ff^{\operatorname{iv}}) - (1 + \lambda_{2})} + \gamma_{1} + \gamma_{2} + \gamma_{2
```

(15)

(14)

(16)

(17)

```
\theta_{1}^{\rho ime \prime } \left( \{1 + \frac{4}{3}R\} \right) + \Pr (f \cdot \{1\}^{\rho ime } - 2f^{\rho ime } \cdot \{1\} ) + \frac{2\lambda \{0\}}{2\lambda \prime } - 2f^{\rho ime } \cdot \{1\} ) + \frac{2\lambda \{0\}}{2\lambda \prime } - 2f^{\rho ime } \cdot \{1\} - 2\lambda \{1\} ) + \frac{2\lambda \{0\}}{2\lambda \prime } - 2f^{\rho ime } \cdot \{1\} - 2\lambda \{1
```

 $\theta_{2}^{\rho \circ \rho \circ } \left(\{1 + \frac{4}{3}R\} \right) + \Pr \left(\{1 + \frac{4}{3}R\} \right) + \Pr \left(\{1 + \frac{4}{3}R\} \right) - 2 \left(\{1 + \frac{1}{3}R\} \right) - 2 \left(\{1 + \frac{1}{3}R\} \right) - 2 \left(\frac{1}{3}R\} \right) -$

The boundary conditions (B.C.) are:

 $$\$ \left\{ \left\{ \left(\frac{1} = 1,\quad \left(\frac{2} = 0\right); for\; PST\; at\; eta = 0 \right) \left(\left(\frac{1} \to 0,\quad \left(\frac{2} \to 0\right); hfill \right) \right\} \right\} \left(\frac{2} \to 0\right) . $$ \left(\frac{2} \to 0$

(18)

"The skin friction coefficient (Cf), local Nusselt number (Nu_{x}) and local Sherwood number (Sh)" are defined as

 $\C_{f} = \frac{(-2)^{y}}{{rho u_{w}^{2}}},\xy = q_{r} - \frac{(xq_{w})^{y}}{{k(T_{c} - T_{w})}}$

(19)

Here

```
$$ \tau_{w} = \left. {\left[ {\mu \left( {\frac{\pi (\pi (\pi (\pi u)})} \pi)} \right)} \right|_{y = 0} ,\;q_{w} = \left. { - \left( {\frac{\pi (\pi (\pi u)})} \pi)} \right|_{y = 0} ;\;{\text{surface}}\;{\text{flux}} $$
(20)
```

On substituting value from Eq. $(\underline{20})$ in to Eq. $(\underline{19})$, we get the following dimensionless expressions for skin friction coefficient, local Nusselt number and local Sherwood number as given below:

```
\ c_{f} {\text{Re}}_{x}^{\frac{1}{2}} = -2f^{\text{prime prime}}  (21)
```

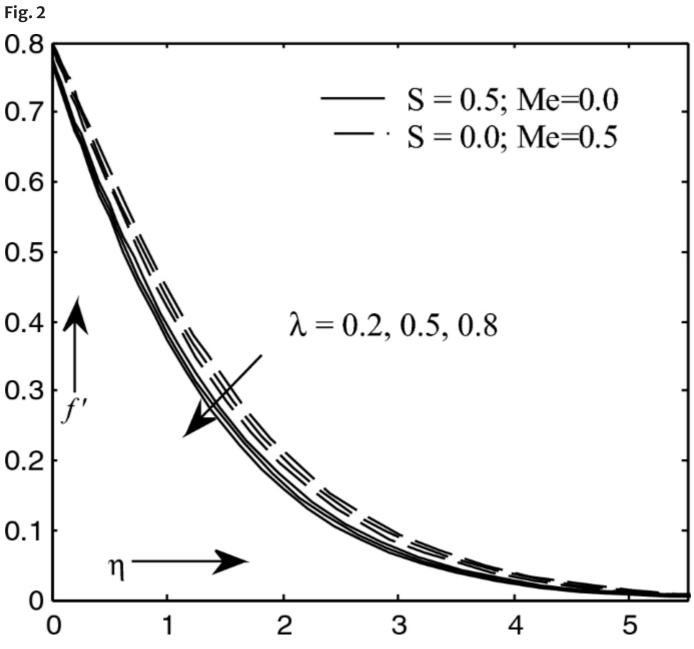
 $\label{eq:continuous} $$ (Sh/\operatorname{Re}) = -\phi^{\circ}(0),\) where ({\text{Re}} = ax^{2}/v) local Reynolds number.$

Result Discussion

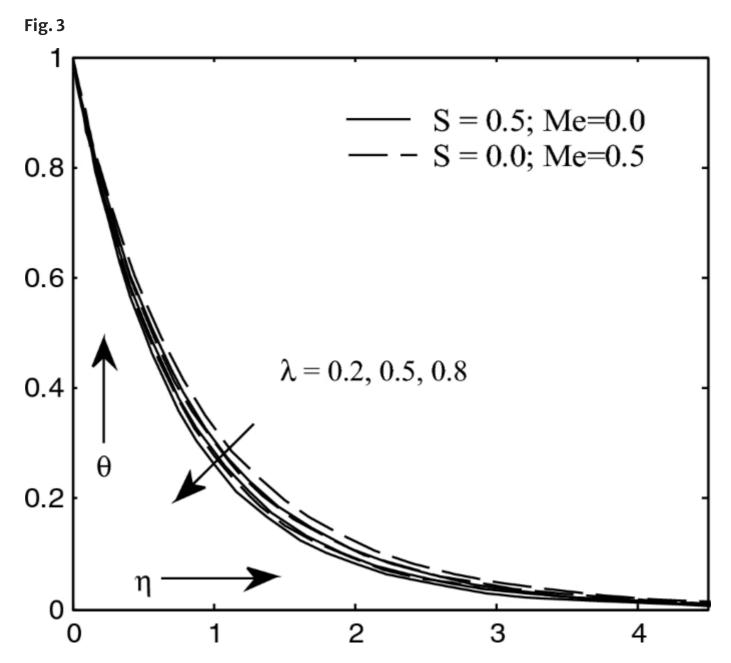
(22)

In the current work, approximate solutions for the melting and permeable flows of a Jeffrey fluid over a linearly slip stretchysurface have been obtained by using R-K 4thorder shooting techniques. The motive of this object was to examine heat transfer and first-order slipy Jeffery fluid flow with magnetic dipole effect. We also investigated radiation with heat sources affecting past permeable linearly stretching or melting sheet. Various sets of the numerical solution have been accepted out for different mixtures of pertinent parameters namely, Various physical characteristics' effects ferromagnetic interaction parameter (\(\lambda_\)), Deborah number (\(\lambda_\)), Radiation parameter (\(\lambda_\)), Heat sources parameter (\(\lambda_\)), Prandtl number (\(\lambda_\)), suction/injection parameter (\(\lambda_\)), ratio of relaxation to retardation times (\(\lambda_\) (lambda_\) on velocity and temperature

profiles is illustrated graphically with condition prescribed surface temperature (PST). It is also noted that Figs. $\underline{2}$, $\underline{3}$, $\underline{4}$, $\underline{5}$, $\underline{6}$, $\underline{7}$, $\underline{8}$, $\underline{9}$, $\underline{10}$, $\underline{11}$, $\underline{12}$, and $\underline{13}$ show that velocity as well as temperature profile with PST condition. By using the figures and tables, the impression of many relevant parameters is explored. The default parameter values for current work are considered as \(\lambda = 0.5\), \(\rho_{1} = 0.2\),\(\lambda_{1} = 0.1\),\(\rho_{1} = 0.1\),\(\



The influence of \(\lambda\) parameter on velocity



The influence of \(\lambda\) parameter on temperature

Fig. 4

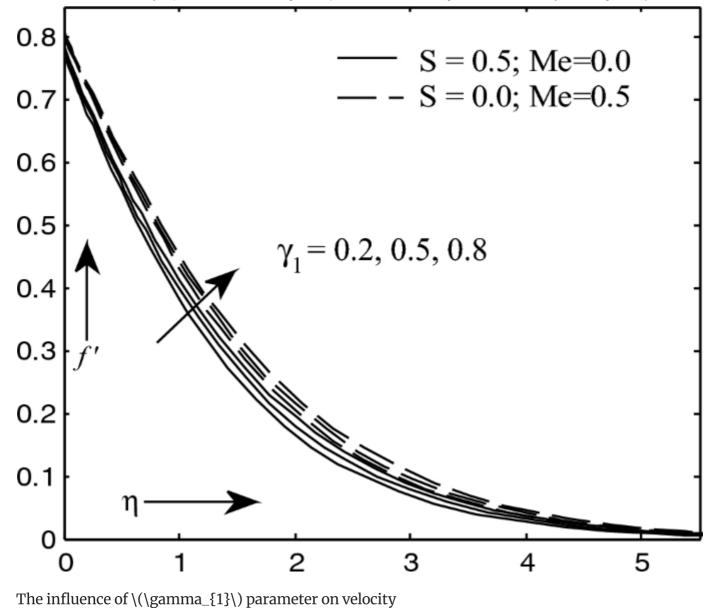
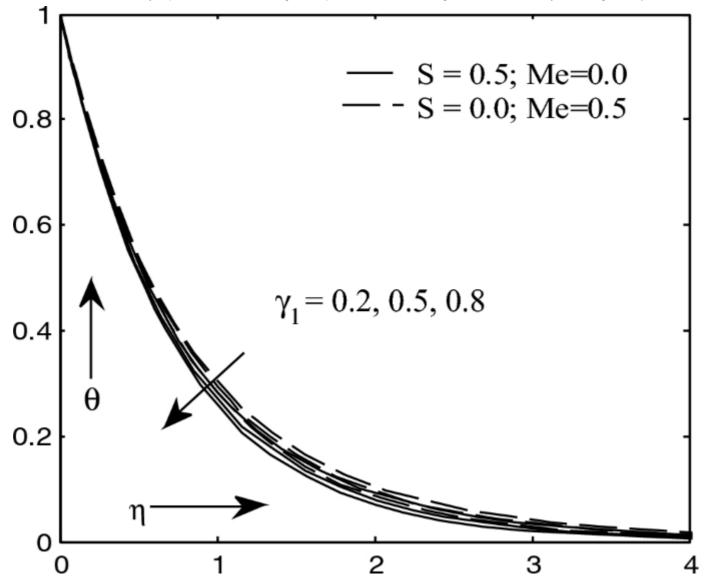


Fig. 5



The influence of (γ_{1}) parameter on temperature

Fig. 6

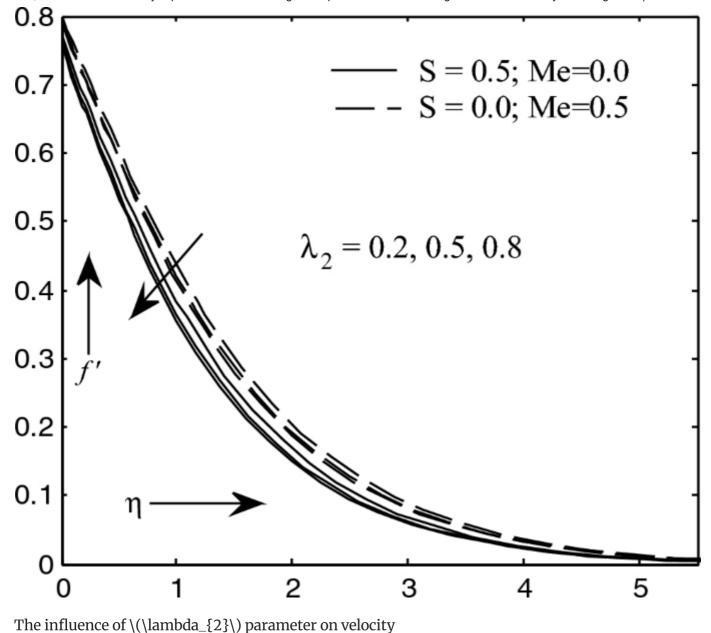
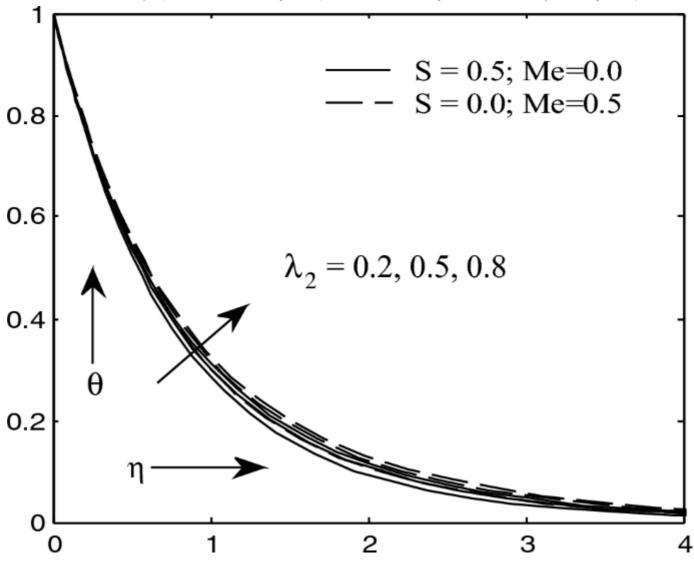


Fig. 7



The influence of $\(\lambda_{2}\)$ parameter on temperature

Fig. 8

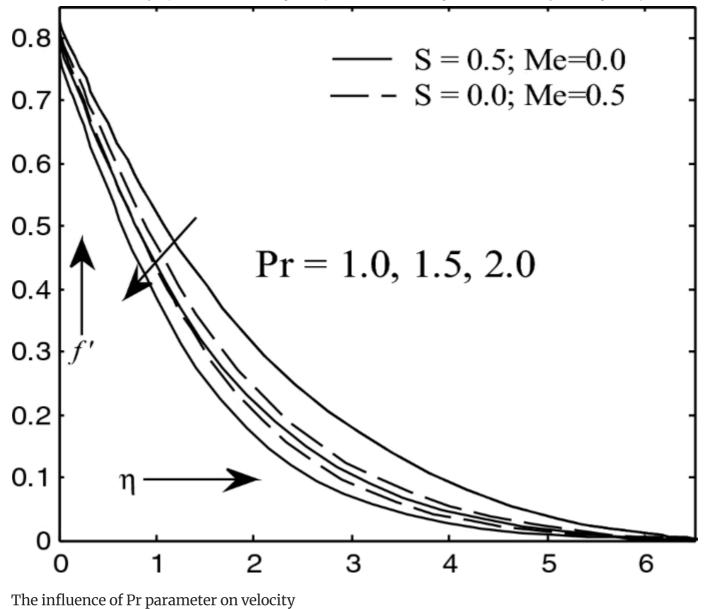
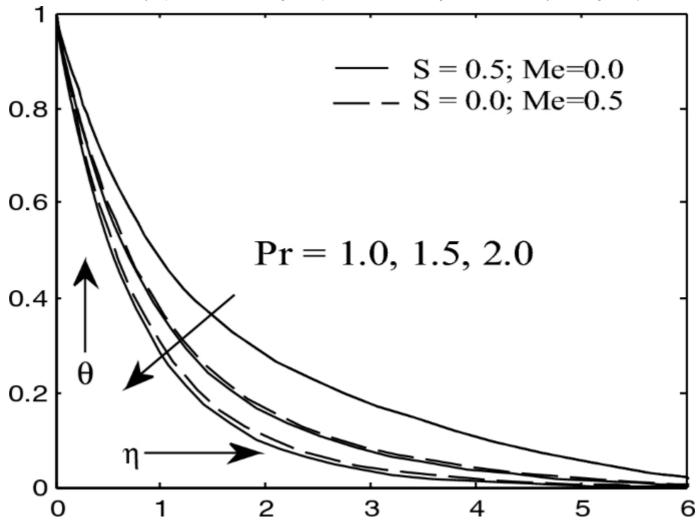


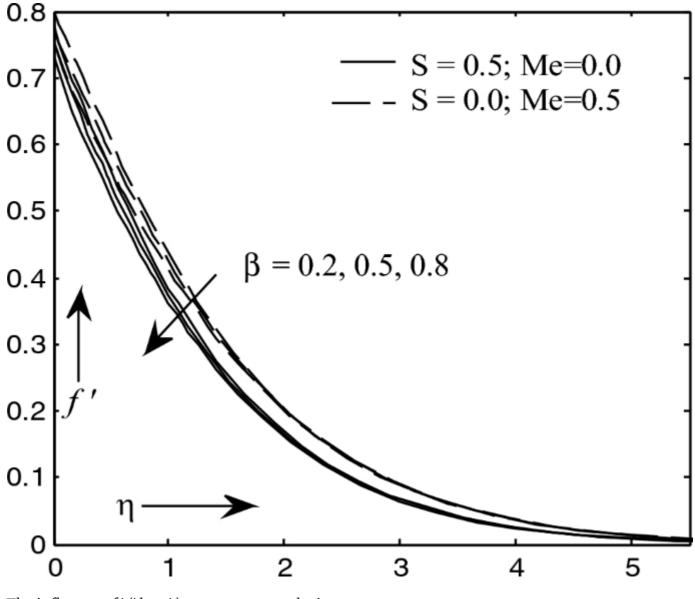
Fig. 9



The influence of Pr parameter on temperature

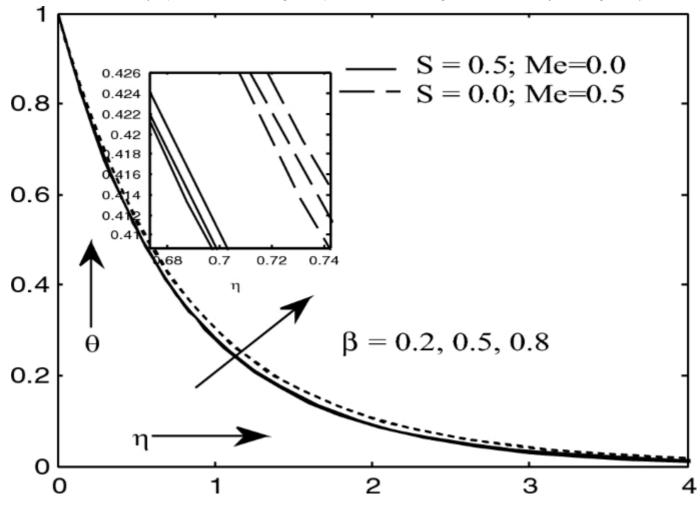
Fig. 10





The influence of $\(\beta\)$ parameter on velocity

Fig. 11



The influence of $\(\beta\)$ parameter on temperature.

Fig. 12

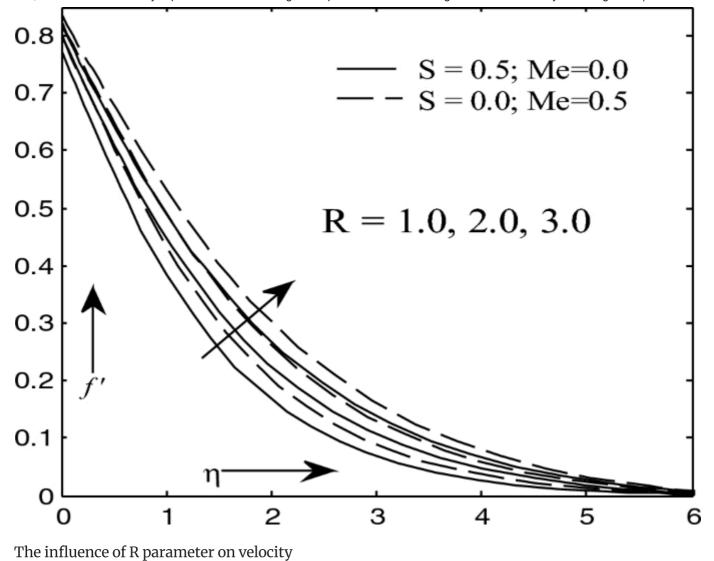
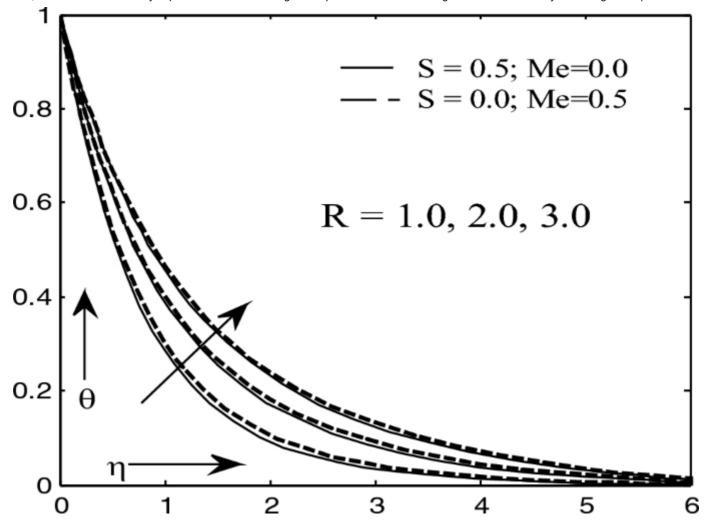


Fig. 13



The influence of R parameter on temperature

Table 1 Comparison of Nusselt number \(- \theta_{1}{\prime} (0)\) for the value of \((\beta\) = 0, \(\lambda_{2}\) = 0, \(R\) = 0, \(\gamma_{1} = S = Q = Gr = 0\)

Table 2 The numerical values of skin friction coefficient (\(Cf\)), Nusselt number (\((Nu\))) in PST surface temperature is given for numerical solution on permeable surface case for various values of the physical parameter

Table 3 The numerical values of skin friction coefficient, Nusselt number in PST is given for numerical solution on melting surface case for various values of the physical parameter

The impression of the viscous dissipation parameter(\(\lambda\)) on temperature distribution as well as velocitycan be discovered in Figs. 2 and 3. It is noticed that velocity and temperature profiles get lower with the growth in the result \(\lambda\). Physically, viscous dissipation occurs because the Jeffery fluid exhibits viscoelastic behavior, meaning that it possesses both viscous and elastic properties. As the fluid flows, the shear forces between adjacent layers cause energy to be transferred from the macroscopic motion of the fluid to the microscopic motion of its molecules. This energy is then dissolute as heat. The dissipation of energy results in a loss of kinetic energy within the fluid, leading to a decrement in velocity. Additionally, as energy is converted to heat, the fluid temperature increases. Consequently, both the velocity together with temperature profiles decrease as a conclusion of viscous dissipation.

Figures 4 and 5 exemplify the outcomes of Deborah's numbers \(\gamma_{1}\) on velocity \(f^{\prime} \left(\eta \right)\), with temperature \(\theta \left(\eta \right)\) profile. Whenever values of \(\gamma_{1}\) gets increased the velocity gets enhanced but alternatively temperature profile gets cut down. Physically, when a Jeffery fluid flows, it experiences viscous dissipation, which leads to the transformation of mechanical energy into heat due to internal friction within the fluid. This dissipation is more prominent at higher Deborah numbers when the elastic effects become significant. The fluid exhibits an increasingly noticeable elastic behavior as the Deborah number rises. This elastic behavior is associated with energy storage and release within the fluid, causing the temperature to be lower.

Figures $\underline{6}$ and $\underline{7}$ exhibit consequences of porosity parameter (\(\lambda_{2}\\)) on velocity \((f^{\perp \text{ right}}\), with temperature \(\text{ theta \left(\eta \right)}\) profile. Figure $\underline{6}$ showsflow stream reduces with improving meritsof the (\(\lambda_{2}\\)) parameter and on the other side effect seenon the temperature profile. Physically, the fluid flow is significantly announced with a growth in relaxation time (or decrease in retardation time) because growth in \(\lambda_{2}\) leads to rise in

relaxation time; it means particles need much time to come back from the perturbed system to equilibrium system in which subsequently fluid velocity get lower.

Figures $\underline{8}$ and $\underline{9}$ show the impression of Prandtl number (\(\Pr\)) on velocity and temperature profiles. Increase the \(\Pr\), and suppress the fluid velocity and thermal energy boundary layer thickness.\(\Pr\) is the ratio of momentum diffusivity to thermal diffusivity. Heat will diffuse from the sheet more quickly from fluids with lower in Pr because they have high-level thermal conductivities.\(\Pr\) may speed up the cooling process in conducting flows. The velocity together with the temperature profile plotted against the similarity variable (\(\end{eta})\) for distinct characteristics of the ferromagnetic interaction parameter(\(\end{eta})\)) parameter is shown in Figs. $\underline{10}$ and $\underline{11}$. The figures indicate that with the rise in the parameter, the thickness of the momentum boundary layer decreases, while the temperature shows the opposite impact.

Figures $\underline{12}$ and $\underline{13}$ exemplify the significance of radiation parameter (\((R\))) on velocity and temperature profile. From these graphs, it is noticeable that the momentum and thermal boundary layer thickness grows up with growth in the results of \((R\)). In general, as increases, the mean absorption coefficient falls, which causes the radiative heat flux to diverge. Resultant, the range of radiative heat transfer to the fluid rises, raising the fluid's temperature.

Conclusion

In the latest work, by utilizing R–K 4th order techniques approximate numerical results for the melting and permeable flow of Jeffrey fluid aroundstretchysurfaces have been derived. The motive of the research was to examineheat–transfer and first–order slipy Jeffery fluid flow with magnetic dipole effect. The investigation also revealed that the viscous dissipation parameter \(\lambda\) had counterintuitive effects on the thermal profile. Deborah numbers \(\gamma_{1}\) on velocity \(f^{\t} \prime \text{ \left(\eta \right)\), with temperature \((\text{\text{theta}} \left(\eta \right)\)) profile. Whenever values of \((\gamma_{1}\)) gets increased the velocity gets enhanced but alternatively temperature profile gets cut down. In this article we find the tabulated form the numerical values of skin friction coefficient, Nusselt number in PST is given for numerical solution on melting surface case and without melting case for various values of the physical parameter. The two terms of flow have been explored. The impression of abundant quantities on velocity with temperature distribution is outlined as follows:

- The velocity $(f^{\rho}) \leq \left(\cdot \right)$ together with temperature $(\theta \in \theta \in \theta)$ and $\theta \in \theta \in \theta$ (\Pr\).
- The velocity \(f^{\prime } \left(\eta \right)\) together with temperature \(\\text{theta} \left(\eta \right)\) profile is higher for the melting boundary condition than permeable boundary condition.
- Meganetic field parameter and porosity parameter have propensity to reduce the skin friction coefficient (\((Cf\))) and local Nusselt number (\((Nu\))).
- Radiation parameter (\(R\)) has propensity to stand up the skin friction coefficient and local Nusselt number.

Data availability

This paper data not published any journal.

Abbreviations

Distance

a:

```
c: Stretching rate ((\{\text{text}\{s\}\} \land \{-1\}))
```

 (C_{p}) : Specific heat at constant pressure

 (C_{f}) : Skin friction coefficient

(f): Dimensionless stream function

 $\(H\): Magnetic field ((\{\text{A}\}/{\text{m}}))$

(k): Thermal conductivity

 (K^{*}) : Pyro magnetic coefficient

```
\(M\): Magnetization ((\{\text{A}\}/{\text{m}}))
\(\kappa\): Extra stress tensor
(N_{ux}): Local Nusselt number
(\Pr = \frac{{mu C_{p}}}{k}): Prandtl number
      Grashof number
Gr:
\(\theta\): Dimensionless temperature
\{\{u_{w}\}\} = \{u_{w}\}\} \{v\} \}: Local Reynolds number
({\text{Ext}}{R})_{1}): Rivlin-Ericksen tensor
(S = \frac{{ - V_{w} }}{{ cv} }): Suction/injection parameter
(R = \frac{4 \cdot T_{\star}}{4 \cdot T_{\star}}^{3}}{3kk^{*}}): Radiation parameter
T:
      Temperature (K)
(Q = \frac{Q^*}{c \cdot p}): Heat sources
(T_{c}): Curie temperature (K)
((u,v)): Velocity components ((\{\text{text}\{ms\}\} \land \{-1\}))
((x,y)): Coordinates along and normal to the sheet ((\{\text{text}\{m\}\}))
(\mu): Dynamic viscosity ((\{\text{N}}), \{\text{ms}}^{-1}))
\( \mu_{0} \): Magnetic permeability
```

```
\(\alpha = a\sqrt\{\frac\{c\}\{v\}\}\):  Dimensionless distance
```

```
\(\gamma_{1} = \lambda_{1} c): Deborah numbers
```

```
\(\rho\): Density ((\{\text\{kg\}\}\;\{\text\{m\}\}\land \{-3\})\)
```

((xi, eta)): Dimensionless coordinate

 $\(\psi\): Stream function \((\{\text{m}}^{2} \; \{\text{s}}^{-1})\)$

 $\(\phi\): Magnetic potential$

\(\tau\): Cauchy stress tensor

 $\label{lem:lembda} $$ (\lambda = \frac{c} - T_{w})} \): Viscous dissipation parameter$

 $(\lambda_{1} \ \ Material parameters of Jeffrey's fluid$

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Contributions

All author contribution to solve the problem and graphically represent. All author verify and examine the result and formulation.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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