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Elasticity modelling of price-based demand response programs considering customer's different behavioural patterns

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Abstract

Mathematical modelling of demand response programs (DRPs) aids regulators and policymakers in analysing the advantages of price-elastic loads on distribution system and electricity market. This mathematical model is used to find updated demand after DR for calculation of different technical and economical indices. In accordance with the concept of price elasticity of demand, an economic model of price sensitive load is developed. Price elasticity model (PEM) is a very useful tool to assess customer participation in DRPs. In this work PEM has been attributed on the ground of price elasticity of demand where PEM has been modelled by means of analytical and stochastic elasticity approach. Analytical elasticity has been modelled on the basis of ideal (lossless) approach whereas Stochastic elasticity has been modelled using Ornstein–Uhlenbeck process based on load flexibility and results obtained from these approaches were compared to check the acceptability of DR programs. It is observed that the proposed APEM and SPEM approaches provides maximum peak curtailment of 15% and 10.2% respectively during peak hours. Similarly, it is also noticed that both approaches improve load factor by reducing its peak-to-valley span by 47.33% and 35.7% respectively. The proposed models are investigated on standard IEEE 33-bus distribution system and modified IEEE 33-bus distribution system and are compared with already existing DR models. Technical and economical comparison of proposed approaches on both distribution systems shows that proposed models are competent enough to model customer behaviour properly.

Previous

Next

Keywords

Demand response; Price elasticity model; Price elasticity of demand; Stochastic elasticity

Nomenclature

Functions and Variables

$\xi(t)$
: Elasticity at time t

B
: Customer benefit function

$D_i^k(t)$
: ADR demand of ith customer of kth class at tth time

$D_{i,o}^k(t)$
: BDR demand of ith customer of kth class at tth time

L
: Lagrange function

N
: Normal distribution

NB
: Net benefit function

$P^k(t)$
: RTP offered to class k at time t

W
: Wiener/Brownian variable

$P_o{}^k(t)$
: RTP offered to class k at time t

Indices and Sets

i, I

Index and set of total customer numbers

k, K

Index and set of different customer classes

t, t', T

Index and set of time instances

$T_{1,2,3}$

Time set of peak, off-peak and valley hours respectively

Parameters and Constants

γ_i^k

Sensitivity coefficient of ith customer of kth class

λ_i^k

Lagrange multiplier of ith customer of kth class

μ, σ, θ

Mean value, degree of volatility and decay/growth rate in O-U process

ρ^k

Subsidy/charging coefficient

1. Introduction

Demand Response (DR) is among the many programs of Demand Side Management (DSM) which deals with the techniques that encourage consumers to monitor and optimise their energy consumption pattern. There are two way benefits from DSM programs; first, consumers may get financial benefits by reducing their electricity bills. Second, the utility may be relieved of burden due to shifting of load demand from peak to non-peak hours [1]. Participation in DRPs having modified pricing mechanism affects customer's overall billing and their comfort simultaneously. However, the implementation of DRPs in the energy system not only provides some benefits but also poses some challenges. These benefits can only be realised after overcoming challenges such as willing participation from consumer side, social, economic, and technological barriers and so on [2]. With the evolution of smart grid, Internet of things (IOT) and modern communication infrastructure, DR adaptation has become a reality as it provides consumers a reality check on their consumption habit and billing estimates. Consumer can be an active element in electricity market by having dynamic pricing, financial incentives and electricity trading options [3]. DRPs are specifically categorised into two groups, Price-based demand response (PBDR) programs and incentive-based demand response (IBDR) programs. These programs are designed to offer several DR options in which a consumer can enrol as per his suitability and socio-economic status [4].

DR is a useful approach which provide sophisticated solution to the existing competitive problems and allow consumers to balance out inequalities between demand and supply. To participate in DR program, proper formulation and mathematical modelling needs to be carried out at regulatory stage and then the perks and incentives of participation in these program are to be conveyed to the consumes. Apart from social and economic constraints, customer's participation depends a lot on their preferences and their usage pattern [5]. This behavioural usage pattern needs to be studied as it plays pivotal role in successful adoption of DR program. Customer's preferences can be modelled as electricity unit categories, unit wise price range and delivery or execution time of the day. DR abstracts its core idea from the area of economics as that deals with the commodity unit, its price & delivery time and maintains balance between these terms. In this work preferences have been modelled on the basis of time span of shifting of electricity usage at which the consumer is satisfied and participating willingly. For this preference modelling, PEM has been formulated for different behaviour patterns and corresponding techno-economic details have been presented in detail.

1.1. Literature review

PEM is a versatile tool that represents the hidden flexibility of customers. This flexibility is very much necessary for successful adoption of DR as it defines the limits for acceptability of DR program and adjustability of load demand. PEM has been vastly discussed in [6] where author discussed economic load modelling for single and multi-period elastic load and simultaneously elaborated DR portfolio sorting based on Technique for Order Preference by Similarity to Ideal solution (TOPSIS) and Analytical Hierarchy Process (AHP). The basis for program prioritising were some technical and economical indices i.e. peak reduction, load factor, peak to valley distance and total cost of customer. In [7], author conceptualised self and cross-elasticity for PEM and applied these elasticity concepts for different participation levels of responsive load and calculated variable price for PBDR programs and incentives for IBDR programs from load factor and saved energy point of view. Several linear and non-linear economic models of price responsive loads such as Power, Exponential and Logarithmic models have been presented in [8] for time-of-use (TOU) program and results obtained from these non-linear models are used to find out conservativeness of the models with reference to original linear model. Results also show that all models shows identical behaviour for small value of elasticity and demand does not change from the base case substantially but as elasticity increases, all non-linear models show different behaviour and demand ratio decreases from its initial value. This work is further extended for IBDR program in [9] where different cases pertaining to variable elasticity, pricing and DR potential is considered for price responsive load and customer benefit function.

A cumulative modelling of IBDR in addition to PBDR for TOU program is done in [10] to eliminate those peaks that remain after load shift due to dynamic price based on demand–price elasticity concept and results showed that both load serving entity (LSE) and customer benefited economically and average value of monthly locational marginal price (LMP) gets reduced. Author suggested that TOU of PBDR may appear as a punishment due to less load shifting from peak to off-peak hour whereas dynamic pricing is applicable to full load cycle. In contrast to this, IBDR sounds attractive due to its rewarding nature and as very less amount of load is to be shifted during occasional peak hour, customer's comfort is not affected and simultaneously billing criteria is not much changed. In [11], the price elasticity concept is constrained to appliance level in place of utility level for IBDR programs and findings suggested that incentive value is less for small power consuming and high elastic load such as lighting, washing and dishwashers whereas it is high for high power consuming and less elastic load i.e. HVAC loads. It is also evident from the results that as the power consumption increased during winter season, average elasticity decreases. [5] developed an electricity and incentive pricing scheme that assures customer's comfort and network elasticity while satisfying uncertainties and inter-temporal constraints. Study was conducted at transmission level and results proved that system behaviour depends on the time range of customer's rationality as well as degree of elasticity.

With the advancement of time, electricity market has become liberalised and customer can choose their demand pattern as per their needs and economy based on self and cross-elasticity and spot price of electricity. In [12], authors explored the relationship between short term elasticity of demand and scheduled generation while setting prices. As per author, demand of any commodity deceases when price of the commodity unit increases and vice-versa, this concept of finance and economics is well illustrated in paper for short term study on a day (24 h) basis. In this work author proposed different kind of customer preferences based on their consumption pattern/habit and convenience. Flexible, postponing, anticipating, inflexible and optimising kind of customers are well defined based on PEM formation which clarify the possibility of load shifting from peak to off-peak or valley period. In [13], author assumed that the customer is consistent in nature i.e. customer's load shifting results not only in load reduction during peak periods but also in load increment during off-peak/valley period. It is clear that a short term dip in price would nor trigger customer to increase their demand as compared to short term hike in price for that particular duration. Customer's behaviour will be different for above two situations and this is due the variable and volatile elasticity of customer. A more elastic demand may sometimes reduce the profit of generation companies [14] but it is practically not possible as generation companies would always earn their profit in terms of revenue earned from customer's billing. A decision framework capsulating stochastic nature of electricity retailer is proposed in [15] where electricity rates for customers based on TOU pricing scheme are defined and risks associated with revenue due to volatile and uncertain nature of pool prices are measured with conditional value at risk (CVaR) methodology. For modelling of uncertainties in pool price and retailer's demand autoregressive moving average model (ARMA) approach is considered.

Most of the study conducted on distribution system deals with short term and long term basis. Similar study is conducted in [16] where an optimising time-series model applicable for few months to few years is proposed based on TOU pricing having uncertainty in pool price and load demand having objective of maximising distribution company (DisCo's) profit. It is illustrated from the study that as DR participation increases, volatility of pool price decreases and correspondingly share of forward market purchase decreases. DR has its impact not only on the DisCos, LSE/utility but also on various specific sectors such as residential, industrial, commercial and agricultural etc. Conclusive study on residential sector having different appliance category for different weather conditions on per day basis is performed in [17] where on the basis of acceptable delay time (ADT) and penetration level of active customers DR potential is utilised. Study demonstrated that as customer participation increases so is the service reliability and overall economy. It is also found out that all household appliances do not have same flexibility (in the case of freezers and refrigerators) as long-time interruption in their service can cause spoilage. In [18], an optimal scheduling pattern for smart home considering PBDR & IBDR and privacy protection is proposed for smart homes where cost of electricity consumption is minimised using Beetle Antennae Search algorithm and privacy of customer is protected using privacy quantification method. Similar appliance based study is proposed in [19] where household appliances are considered to find a trade-off between cost efficiency and consumer's preference for residential load in smart grid. Residential DR is further reviewed in [20], [21], [22] using latest information and communication technologies (ICT) in relevant smart grid applications. In [23], a new bi-level pricing scheme for retailers and consumers of multi-energy buildings is proposed for PBDR using Dantzig–Wolfe decomposition. It maximises retailer's day ahead expected profit and also maximises the DR capacity with proper coordination between multiple buildings.

An adaptive economic framework based on PEM for PBDR program covering deterministic and stochastic elasticity is modelled in [24] where deterministic elasticity is calculated on the basis of lossless DR adaptation and stochastic elasticity is modelled by using Geometric Brownian motion (GBM) phenomenon. Results showed that various classes i.e. residential, commercial, industrial and agricultural classes follow load pattern of each class closely and load recovery in off-peak/valley period is accompanied with dynamism in elasticity in cross periods. In stochastic modelling customer's behaviour is assumed to be rational but uncertain in nature. Load recovery phenomenon in cross-elastic periods is also taken into consideration in [25] which represents the inter-temporal impact of load recovery in scheduling of DR. Concept of securitisation is well illustrated in this paper according to which cost and benefit associated with amount of load recovery follow exponentially decaying curve as discount offered to customers vary according to Brownian motion. In [26], PEM is again modelled for IBDR program where the augmented value of incentives is calculated and a detailed comparison of several programs is carried out on IEEE 33 bus system and it is concluded that demand sensitivity follows IBDR, PBDR and a combination of IBDR & PBDR in ascending order. Similar work has been done in [27] where author assumed dynamic elasticity for estimation of load profile based on stochastic nature of customer in contemporary distribution system. Author also proposed several indices related to technical assessment in the interest of stakeholders.

A smart distribution system with price responsive and controllable load in three-phase unbalanced distribution system is proposed in [28] where demand is exponentially dependent on price and a novel constant energy load model is proposed. In [29], a Stackelberg game approach is proved to be an effective and efficient method for RTP based DR for load control between retailer and devices. RTP is again well illustrated in [30] where price volatility of power grid is defined in terms of maximal relative price elasticity which is defined as the maximal ratio of generalised price elasticity of consumers to that of producers. In [31], authors proposed several possibility of elasticity matrices based on load shifting nature of customers. In [32], an improved IBDR program for day-ahead and intra-day market is proposed where the concept of improved elasticity is illustrated which depends not only on electricity price but also on the consumption hour as well as customer type. A DR framework based on RTP calculation is proposed in the case of system contingency (outage in generation unit or transmission line) and the role of DR is investigated for having economic and secure network [33]. Based on the findings it is evident that proposed load model results in more uniform system load profile, reduction in net energy drawn and less feeder losses. In PEM self and cross-elasticity govern the shifting and adjustment of load demand from peak period to off-peak/valley period so modelling of DR program based on PEM is indispensable work that should account dynamic or variable price, stochastic nature of customers' load demand and other inter-temporal constraints.

1.2. Motivation and contribution

Taking into consideration the above literature review, it is clear that PEM is a promising solution to model relationship between customer's demand and pricing scheme. It defines customer's acceptability of DR approach based on dynamic price for PBDR and incentive/penalty for IBDR. As the customer's demand pattern is variable throughout the day, a single and constant value of elasticity will not serve the purpose of load recovery and load modelling. Though this elasticity can be constant and presumed in case of simple PEM modelling and deterministic PEM modelling but these two modelling are amateur and ideal in nature respectively. That is why stochastic modelling is the need of the hour and suits well as per the non-linear nature of load demand. In stochastic PEM, the cross-elasticity is dispersed randomly throughout the day based on self-elasticity and applicable mechanism. It justifies the interrelation between random nature of load variation and dynamic price as both are time dependent. Most of the DR study is conducted on utility or distribution system level considering utility indices as main parameters and constraints, less attention has been paid on the individual customer classes such as residential, industrial and so on.

In view of this, the proposed work presents a flexible methodology that simulates a relationship between dynamic price and load demand based on dynamic price elasticity of demand. This dynamic elasticity is obtained using two approaches namely: analytical elasticity and stochastic elasticity. Analytical elasticity sets up relationship between peak period elasticity and cross-period elasticity based on lossless DR phenomenon where dynamic nature of elasticity for cross period is obtained via suitable self-elasticity maintaining overall consumption before DR (BDR) and after (ADR) same. In stochastic elasticity, Ornstein–Uhlenbeck process is used to provide dynamism for cross elastic periods so that load recovery can be modelled over the time while satisfying inter-temporal constraints for flexible (shiftable/deferrable and curtailable) loads. Moreover this stochastic behaviour of customers is well illustrated deeply by assuming customers to have anticipating and postponing nature alongwith flexible nature in shifting demand from peak to off-peak periods. So proper analysis is carried out in results of before and after DR so that clear outcome of this variation in customer's nature can be investigated. The main contributions of the paper are summarised as below:

- A comprehensive mathematical modelling framework based on dynamic price elasticity of demand to set up an interrelation between variable & stochastic load demand and dynamic pricing scheme is proposed to make DR an efficient means of load adjustment.
- Dynamic elasticity is modelled via analytical and stochastic approach. In analytical approach, complete load recovery is obtained between peak demand period and non-peak demand periods for shiftable loads on the assumption of ideal/lossless DR.
- In stochastic approach, dynamism in elasticity is modelled using Ornstein–Uhlenbeck process to illustrate stochastic nature of customers for shiftable loads. The essence of stochastic approach is further examined for anticipating and postponing customers.
- The proposed methodology is examined for different customer classes to model diversification in load patterns. Several techno-economy indices are calculated in order to compare the before and after DR effect on system's performance technically and economically.

The remaining paper is planned as follows. Section 2 describes proposed methodology. Section 3 consists of load generation and profiling. Results and techno-economic comparison is done in Section 4. Finally, Section 5 concludes the paper.

2. Economic modelling of DR framework

In the proposed framework relation between dynamic price and load demand is set up by means of price elasticity of demand (PED). PED is calculated as the ratio of differential change in demand as a result of differential change in price. Mathematically it is defined as

$$\xi = \left(\frac{D_{final} - D_{initial}}{P_{final} - P_{initial}}\right)\left(\frac{P_{initial}}{D_{initial}}\right) \tag{1}$$

For two different time frames, Eq. (1) can be written as

$$\xi(t, t') = \left(\frac{\Delta D(t,t')}{\Delta P(t,t')}\right)\left(\frac{P_o}{D_o}\right) \tag{2}$$

$$\begin{cases} \xi(t,t') \text{ is negative for same time instance } (t = t') \\ \xi(t,t') \text{ is positive for different time instances } (t \neq t') \end{cases} \tag{3}$$

Eq. (3) defines the nature of PED in the terms of self-elasticity and cross-elasticity. Self-elasticity is defined for when the demand change occurs in response to the price change for the same time period (t). It is negative in nature as demand generally decreases for increase in price and this concept is valid for single-period sensitivity load i.e. lightening load as they cannot be shifted to other time. Cross-elasticity defines the change in power consumed for one period (t) in response to price change in other period (t'). It is positive in nature and valid for multi-period elasticity loads i.e. dishwashers & washing machines etc. as they can be shifted to other time period. It is worth mentioning here that although the value of self-elasticity is negative but it is way larger than cross-elasticity. It is due to the reason that any change in price will affect demand of the same time mostly whereas its impact will be lesser at cross periods but that response can be scattered throughout the time horizon as it is not possible that whole cross-period impact on demand will be visible at one time slot. It is also one of the reasons of scattered behaviour and small value of cross-elasticity.

2.1. Price elasticity model (PEM)

Assuming a customer to be an active participant of DR program, let us assume that the customer changes his demand consumption in response to price change, then customer's net benefit can be defined as the difference of benefit earned by customer from consuming electricity and payment for that particular amount of electricity. So customer's net benefit function is given by [24]

$$NB\left(D_i^k(t)\right) = B\left(D_i^k(t)\right) - D_i^k(t)\, P^k(t) \tag{4}$$

where, $NB\left(D_i^k(t)\right)$ and $B\left(D_i^k(t)\right)$ are defined as net benefit function and benefit function after consuming $D_i^k(t)$ amount of electricity for ith customer of kth class. The dynamic price offered to any customer of kth class is denoted by P^k. It is assumed that the customer's response is rational and dependent upon the benefits earned by consuming electricity and in line with inter-temporal constraints such as keeping overall consumption of electricity same before DR and after DR. So this energy balance condition can be taken into consideration by multiplying this constraint with Lagrange multiplier λ and adding with objective function. The Lagrange function is given by [34]

$$L\left(D_i^k(t)\right) = NB\left(D_i^k(t)\right) + \lambda_i^k\left\{\sum_{t\in T_1} \Delta D_i^k(t) - \sum_{t\in\{T_2\cup T_3\}} \Delta D_i^k(t)\right\} \tag{5}$$

In Eq. (5), first term represents the net benefit function and second term represents energy balance condition/constraint which is a difference of net change in energy during peak and off-peak/valley period. Putting the value from Eq. (4) into Eq. (5), we get

$$L\left(D_i^k(t)\right) = B\left(D_i^k(t)\right) - D_i^k(t)\,P^k(t) + \lambda_i^k\left\{\sum_{t\in T_1} \Delta D_i^k(t) - \sum_{t\in\{T_2\cup T_3\}} \Delta D_i^k(t)\right\} \tag{6}$$

Simplifying Eq. (6), we have detailed Lagrange function as given

$$L\left(D_i^k(t)\right) = B\left(D_i^k(t)\right) - D_i^k(t)\,P^k(t) + \lambda_i^k\left\{\sum_{t\in T_1} \left(D_{i,o}^k(t) - D_i^k(t)\right) - \sum_{t\in\{T_2\cup T_3\}} \left(D_i^k(t) - D_{i,o}^k(t)\right)\right\} \tag{7}$$

Here $D_i^k(t)$ and $D_{i,o}^k(t)$ represents the demand after DR and demand before DR of ith customer of k th class at tth time. Differentiating above equation with respect to $D_i^k(t)$ to get optimal condition for Lagrange function.

$$\frac{\partial L\left(D_i^k(t)\right)}{\partial D_i^k(t)} = \frac{\partial B\left(D_i^k(t)\right)}{\partial D_i^k(t)} - P^k(t) - \lambda_i^k \tag{8}$$

Equating Eq. (8) to zero, we get

$$\frac{\partial B\left(D_i^k(t)\right)}{\partial D_i^k(t)} = P^k(t) + \lambda_i^k \tag{9}$$

The double derivative of Eq. (9) gives

$$\frac{\partial B^2\left(D_i^k(t)\right)}{\partial D_i^k(t)^2} = \frac{\partial P^k(t)}{\partial D_i^k(t)} \tag{10}$$

In terms of elasticity, Eq. (10) can be extended to

$$\frac{\partial B^2\left(D_i^k(t)\right)}{\partial D_i^k(t)^2} = \frac{1}{\xi_i^k(t)}\frac{{P_o}^k(t)}{D_{i,o}^k(t)} \tag{11}$$

Using Taylor's series expansion method for quadratic expansion of customer benefit function,

$$B\left(D_i^k(t)\right) = B\left(D_{i,o}^k(t)\right) + B'\left(D_{i,o}^k(t)\right)\left(D_i^k(t) - D_{i,o}^k(t)\right) + B''\left(D_{i,o}^k(t)\right)\frac{\left(D_i^k(t)-D_{i,o}^k(t)\right)^2}{2} \tag{12}$$

Putting values from Eqs. (9), (11) into Eq. (12), we get

$$B\left(D_i^k(t)\right) = B\left(D_{i,o}^k(t)\right) + \left({P_o}^k(t) + \lambda_i^k\right)\left(D_i^k(t) - D_{i,o}^k(t)\right) + \frac{{P_o}^k(t)}{\xi_i^k(t)D_{i,o}^k(t)}\frac{\left(D_i^k(t)-D_{i,o}^k(t)\right)^2}{2} \tag{13}$$

Differentiating Eq. (13) and putting it equal to Eq. (9), we get

$$P^k(t) + \lambda_i^k = {P_o}^k(t)\left\{1 + \frac{\left(D_i^k(t)-D_{i,o}^k(t)\right)}{\xi_i^k(t)D_{i,o}^k(t)}\right\} \tag{14}$$

Simplifying Eq. (14) and getting results in terms of updated load demand after DR

$$D_i^k(t) = D_{i,o}^k(t)\left\{1 + \xi_i^k(t)\,\frac{\left(P^k(t)-{P_o}^k(t)+\lambda_i^k\right)}{{P_o}^k(t)}\right\}\forall t\in T \tag{15}$$

Eq. (15) is valid for self-elastic load as it gives variation in demand in response to variation in price for same time period. For cross-elastic loads Eq. (15) gets converted to

$$D_i^k(t) = D_{i,o}^k(t)\left\{1 + \sum_{t'\in T} \xi_i^k(t,t') \frac{\left(P^k(t') - {P_o}^k(t') + \lambda_i^k\right)}{{P_o}^k(t')}\right\} \forall t, t' \in T \tag{16}$$

Combining the effects of both self and cross-elastic loads, updated load demand after DR can be given as

$$D_i^k(t) = D_{i,o}^k(t)\left\{1 + \xi_i^k(t) \frac{\left(P^k(t) - {P_o}^k(t) + \lambda_i^k\right)}{{P_o}^k(t)} + \sum_{\substack{t'\in T\\ t'\neq t}} \xi_i^k(t,t') \frac{\left(P^k(t') - {P_o}^k(t') + \lambda_i^k\right)}{{P_o}^k(t')}\right\} \forall t, t' \in T \tag{17}$$

Above equation gives demand after DR as a result of dynamic pricing and is a combination of single period and multi period elastic loads. The term self-elasticity and cross-elasticity is well illustrated in Eq. (3), still a clearer illustration of price elasticity matrix needs to be done. For this purpose a price elasticity matrix for 24 h (one-day basis) is shown below,

$$\begin{bmatrix} \Delta D(1) \\ \Delta D(2) \\ \dots \\ \Delta D(24) \end{bmatrix} = \begin{bmatrix} \xi(1,1) & \xi(1,2) & \dots & \xi(1,24) \\ \xi(2,1) & \xi(2,2) & \dots & \xi(2,24) \\ \dots & \dots & \dots & \dots \\ \xi(24,1) & \xi(24,2) & \dots & \xi(24,24) \end{bmatrix} \begin{bmatrix} \Delta P(1) \\ \Delta P(2) \\ \dots \\ \Delta P(24) \end{bmatrix} \tag{18}$$

Eq. (18) defines the relative change in load demand as a result of change in price, this behaviour is applicable for the proposed study of 24 h duration. Diagonal elements of above price elasticity matrix are known as self-elasticity coefficient whereas off-diagonal elements are known as cross-elasticity coefficient. Change in load demand for any particular hour can be obtained by using Eq. (18) as follows:

$$\Delta D(1) = \xi(1,1)\,\Delta P(1) + \xi(1,2)\,\Delta P(2) + \cdots \dots + \xi(1,24)\,\Delta P(24) \tag{19}$$

$$\Delta D(t) = \sum_{t'\in T} \xi(t,t')\,\Delta P(t') \quad \forall t' \in T \tag{20}$$

Eqs. (19), (20) defines the total change in load demand for any particular hour whereas total change in load demand for 24 h can be calculated as

$$\sum_{t\in T} \Delta D(t) = \sum_{\substack{t\in T\\ t'\in T}} \xi(t,t')\,\Delta P(t') \quad \forall t, t' \in T \tag{21}$$

From the above analysis it is clear that price elasticity of demand plays an important role in defining the customer's contribution in DR. It is evident from Eq. (21) that price elasticity of demand and change in load demand are proportional to each other and change in demand depends not only on self-elasticity but also on cross-elasticity. It is important to understand that price elasticity matrix contains several terms and to assess the effect of each individual term is cumbersome and hectic as the customer's response in non-linear and complex in nature. To simplify and to justify the lossless DR and stochastic nature of customer, this work proposes a framework based on analytical PEM and stochastic PEM for load recovery.

2.2. Proposed analytical PEM (APEM)

It is presumed that once a customer participate in DR, there will be load curtailment in peak hour and load shifting in off-peak/valley hour. The nature of load shifting should be from peak to off-peak/valley hour. On the basis of load recovery or energy balance phenomenon, algebraic sum of change in load demand during peak and off-peak/valley hours should be equal to zero, as given in constraint of Eq. (5). Another concept says that for equal redistribution of load or for equality constraint to be valid, Eq. (22) should be satisfied [33].

$$\sum_{\substack{t\in T \\ t'\in T}} \xi\left(t,t'\right)=0 \quad \forall t,t'\in T \tag{22}$$

According to Eq. (22), the sum of elasticity for a column of price elasticity matrix (as shown in Eq. (18)) should be zero. Each element of a column of the elasticity matrix represents change in demand during time horizon due to change in price of a particular hour. It also advocates that change in demand during self-elastic period should be compensated during cross-elastic period. But in practice the actual load recovery depends not only on elasticity variation but also on cumulative effect of load demand and elasticity. So Eq. (22) gets modified to Eq. (23) which represents that the peak hour and non-peak hour multiple of load demand and elasticity should be equal to each other.

$$\xi_i^k\left(t\right)D_{i,o}^k\left(t\right)=\xi_i^k\left(T_1\right)D_{i,o}^k\left(T_1\right) \quad \forall t\in\left(T_2\cup T_3\right) \tag{23}$$

Eq. (23) defines the relationship between peak hour and non-peak hour credentials. This equation is based on the assumption that the value of peak hour elasticity and load demand is well known so the corresponding values of elasticity for off-peak/valley hours can be calculated. With the help of this information an overall price elasticity matrix can be formed which eliminates the need of individual self and cross-elastic elements. In the proposed framework peak hour is defined on the basis of peak price of wholesale market price. It is due to the reason that most customers take decisions on the basis of their monetary preferences. Then first of all customers elasticity at peak hour needs to be calculated then only the elasticity at off-peak/valley hours can be find out. For this purpose it is assumed that a customer's maximum participation is only γ times of his demand at peak hour. As per this assumption, peak hour elasticity can be calculated as:

For $t=T_1$ at which $P_k=max\left(P_k\right)$

$$\Delta D_i^k\left(T_1\right)=\gamma_i^k D_{i,o}^k\left(T_1\right) \tag{24}$$

so, peak hour elasticity can be given by

$$\xi_i^k\left(T_1\right)=\left(\frac{\Delta D_i^k(T_1)}{\Delta P^k(T_1)}\right)\left(\frac{P_o^k}{D_{i,o}^k(T_1)}\right) \tag{25}$$

so, elasticity value for off-peak/ valley period can be calculated as

$$\xi_i^k\left(t\right)=\frac{\xi_i^k(T_1)D_{i,o}^k(T_1)}{D_{i,o}^k(t)} \quad \forall t\in\left(T_2\cup T_3\right) \tag{26}$$

It is worth mentioning that γ_i^k is a sensitivity coefficient that represents ith customer's (of kth class) sensitivity of participating in DR program at the event of maximum price. This parameter varies class-wise and depends upon several other factors such as customer's willingness to participate in DR and his social & financial status etc. Eq. (26) is used to calculate the value of elasticity for off-peak/valley hours so that complete response of peak hour elasticity and cross-peak hour elasticity can be visualised in finding the load demand after DR as follows:

$$D_i^k(t) = D_{i,o}^k(t) + \frac{\xi_i^k(T_1) D_{i,o}^k(T_1)}{D_{i,o}^k(t)} D_{i,o}^k(t) \frac{\left(P^k(t) - {P_o}^k(t) + \lambda_i^k\right)}{{P_o}^k(t)} \qquad \forall t \in T, \forall i \in I \tag{27}$$

Eq. (27) gives updated demand after DR for analytical PEM which is based upon load recovery and energy balance principle. In analytical PEM total energy consumed before and after DR remains same that is why it is also called as ideal or lossless PEM. In this approach elasticity value for off-peak/valley hours are calculated on the basis of peak hour elasticity so it sets up a relationship between peak hour demand and off-peak hour demand so that complete load recovery can be obtained.

2.3. Proposed stochastic PEM (SPEM) using Ornstein–Uhlenbeck process

As of now, the value of elasticity weather it be self-elasticity or cross-elasticity are either constant or dependent upon some mechanism (analytical PEM) as discussed in above section. But these elasticity values are constant in nature which represents static process. To inculcate the virtue of randomness as observed in customer's behaviour, the value of cross-elasticity should be non-constant and time-variant in nature. Due to their inherent characteristics, self-elasticity and cross-elasticity compensate each other and this compensation is the basis of load recovery as demand reduction in peak hours should be build up during off-peak hours. If increase in demand is not up to the expectations then load recovery will not be complete and the system response will be lossy or non-ideal. Eq. (28) describes the demand characteristics of lossy system. In Eq. (28), the first term represents change (decrement) in demand during peak-hours whereas second term represents change (increment) in load demand during off-peak/valley hours.

$$\sum_{t \in T_1} \Delta D_i^k(t) > \sum_{t \in (T_2 \cup T_3)} \Delta D_i^k(t) \tag{28}$$

If demand increase during off-peak hours is greater than demand reduction during peak hours then there are chances of generation of new peak in the system which is not desirable at all. Eq. (29) is applicable for this case where increase in demand during off-peak hours overcompensates reduction in demand during peak hours.

$$\sum_{t \in T_1} \Delta D_i^k(t) < \sum_{t \in (T_2 \cup T_3)} \Delta D_i^k(t) \tag{29}$$

So the shifting of load demand from peak to off-peak hours requires proper coordination between self and cross-elasticity and the values of cross-elasticity should reflect stochasticity as observed in customer's behaviour. Moreover customer's response to dynamic price may be random but this randomness is also finite and must lie within limits. It is assumed throughout in this work that customer's response is rational so the value of cross-elasticity may be time-variant and uncertain but it follows a predetermined path. As cross-elasticity is an indication of demand build-up during off-peak hours satisfying inter-temporal constraints like random behavioural pattern, this stochasticity of customer's behaviour is exhibited in PEM by using a continuous-time stochastic process known as Ornstein–Uhlenbeck (O-U) process [35].

The O–U process is a stochastic process in continuous time with its main application in financial analysis. The process is thought to be a modification of the Wiener process. In contrast to Wiener process, O-U process has a bounded variance and stationary probability distribution. The difference between Wiener process and O-U process lies in their "drift" term. The drift is constant for the Wiener process, but for the O-U process it depends on the current value of the process. The drift is positive if the current value of the process is less than the mean and vice-versa. In other words, the mean serves as the process's equilibrium level which gives rise to the process's descriptive name, "mean-reverting". This mean-reverting property has its advantages due to the inclusion of randomness in walk and simultaneously maintaining limit around its mean value. This satisfies both the essential requirements of cross-elasticity as discussed above. According to O-U process, a stochastic process $\xi(t)$ can be defined by the following differential equation:

$$d\xi(t) = \theta(\mu - \xi(t))\,dt + \sigma dW(t) \tag{30}$$

where, ξ represents the stochastic variable which in our case is the cross-elasticity, μ represents the mean value or equilibrium point, σ represents degree of volatility of cross-elasticity and θ represents the rate (decay/growth) by which cross-elasticity reverts towards its mean value during the process. $W(t)$ denotes the Wiener process. Eq. (30) is sometimes also written as a Langevin equation in the form given as

$$\frac{d\xi(t)}{dt} = \theta(\mu - \xi(t)) + \frac{\sigma dW(t)}{dt} \tag{31}$$

$$\frac{d\xi(t)}{dt} = \theta(\mu - \xi(t)) + \sigma\eta(t) \tag{32}$$

where $\eta(t)$, which is a derivative of Wiener process represents white noise. On solving Eq. (31) with the help of Ito's integral [36], [37], the following expression is obtained.

$$\xi(t) = \xi_0 e^{-\theta t} + \mu\left(1 - e^{-\theta t}\right) + \frac{\sigma}{\sqrt{2\theta}} W\left(1 - e^{-2\theta t}\right) \tag{33}$$

where, ξ_0 is assumed to be the initial value at $t = 0$. The generalised expression is given as

$$\xi(t + \Delta t) = \xi(t)\,e^{-\theta\Delta t} + \mu\left(1 - e^{-\theta\Delta t}\right) + \frac{\sigma}{\sqrt{2\theta}} W\left(1 - e^{-2\theta\Delta t}\right) \tag{34}$$

Since the Ito's integral of deterministic integrand is normally distributed, the complete expression of cross-elasticity as per O-U process after putting the value of Wiener variable is given as follows:

$$\xi(t + \Delta t) = \xi(t)\,e^{-\theta\Delta t} + \mu\left(1 - e^{-\theta\Delta t}\right) + \sigma\sqrt{\frac{1 - e^{-2\theta\Delta t}}{2\theta}} N[0, 1] \tag{35}$$

The above expression represents time varying and stochastic nature of cross-elasticity yet maintaining bounded variance in its values. Here Δt denotes the variation in time which is taken on hourly basis in the given study. Similarly, the generalised expression of cross-elasticity for nth change in Δt is given by Eq. (36).

$$\xi(t + n\Delta t) = \xi(t)\,e^{-n\theta\Delta t} + \mu\left(1 - e^{-n\theta\Delta t}\right) + \sigma\sqrt{\frac{1 - e^{-2n\theta\Delta t}}{2\theta}} N[0, 1] \tag{36}$$

As cross-elasticity has been referred to t' instant through out the work, t and Δt of above equation has been replaced by t' and $\Delta t'$ respectively. Now, combining the self and cross-elasticities, the ADR demand for SPEM approach is given by updating Eq. (17), as given below

$$D_i^{cl}(t) = D_{i,o}^{cl}(t)\left\{1 + \xi_i^{cl}(t)\frac{\left(P^{cl}(t) - P_o{}^{cl}(t) + \lambda_i^{cl}\right)}{P_o{}^{cl}(t)} + \xi_i^{cl}(t,t')\frac{\left(P^{cl}(t') - P_o{}^{cl}(t') + \lambda_i^{cl}\right)}{P_o{}^{cl}(t')} + \sum_{\substack{t' \in T \\ t' \neq t}} \xi_i^{cl}(t, t' + n\Delta t') \times \frac{\left(P^{cl}(t' + n\Delta t') - P_o{}^{cl}(t' + n\Delta t') + \lambda_i^{cl}\right)}{P_o{}^{cl}(t' + n\Delta t')}\right\} \tag{37}$$

Here,

$$n = \begin{cases} 0 & if \quad t' \in T_1 \\ n \in 1, 2, \ldots .24 & if \quad t' \in (T_2 \cup T_3) \end{cases}$$

Eq. (37) consists of four terms, where second, third and fourth term indicates demand change due to self-elasticity, initial cross-elasticity at t' instant and summation of all other cross-elasticities at $(t' + n\Delta t')$ instants. Eq. (36) depicts the stochastic modelling of cross-elasticity based on the customer's behavioural pattern using variable drift and volatility in its formulation. Once the basis for formulation of stochasticity is finalised, the O-U process can be extended for modelling various kind of behavioural patterns of customers i.e. flexible, anticipating and postponing customers. Flexible customers are those customers who can shift their consumption throughout the day so that demand consumption during peak hours can be reduced whereas anticipating type of customers have tendency of preponing the load demand from peak hours to off-peak/ valley hours. Postponing customers tend to delay their consumption cycle in response to dynamic price so that DR objective can be achieved. Fig. 1 denotes the structure of price elasticity matrix for flexible, anticipating and postponing type of customers [31]. These three type of customers are chosen for load modelling under DR in the proposed work.

The diagonal elements of these matrices represent self-elasticities whereas off-diagonal elements correspond to cross-elasticities. Vertical straight lines in these matrices denote non-zero and finite values whereas empty space in matrices is an indication of zero cross-elasticity. This matrix structure has been formulated for 24-hour duration on the basis of following equation: $[\Delta D]=[PEM][\Delta P]$. Any column of this PEM matrix (say jth column) is an indication of demand change during entire 24-hour duration due to change in price at single jth period. It can be seen that flexible customers have the ability to reschedule their demand consumption over a long period that is why non-zero elements are spread widely around the diagonal. Anticipating customers have the probability of bringing forward their consumption that is why PEM structure has finite values above the diagonal in opposite to postponing customers. It is to be noted that if the system have more than one peak then anticipating and postponing customers will have corresponding behaviour as during only one peak in the load curve. Both APEM & SPEM approaches have been illustrated with the help of flow diagram in Fig. 2.

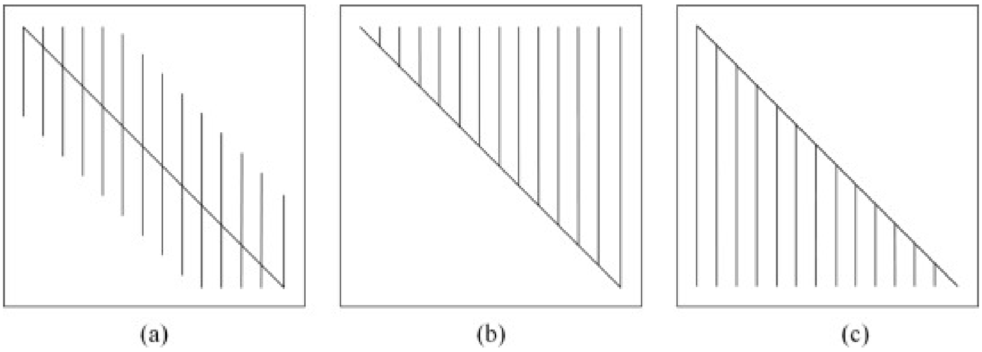

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Fig. 1. PEM structure for various type of customer's behaviour patterns: (a) flexible customers, (b) anticipating customers, (c) postponing customers.

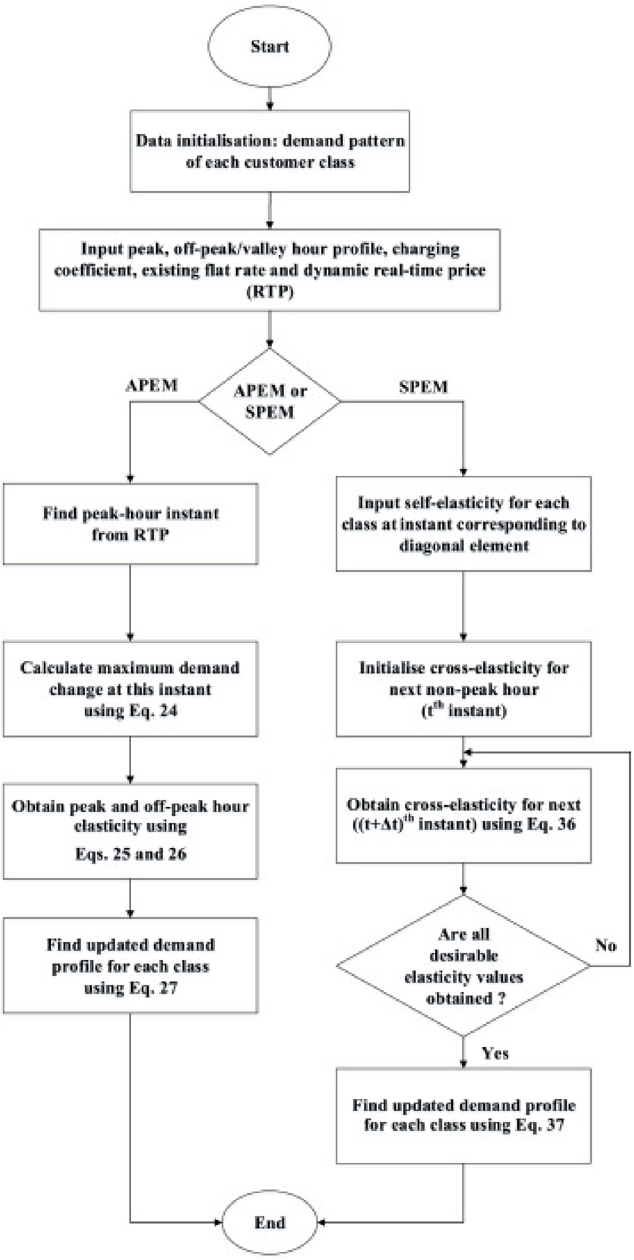

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Fig. 2. Flowchart of proposed APEM & SPEM modelling approach.

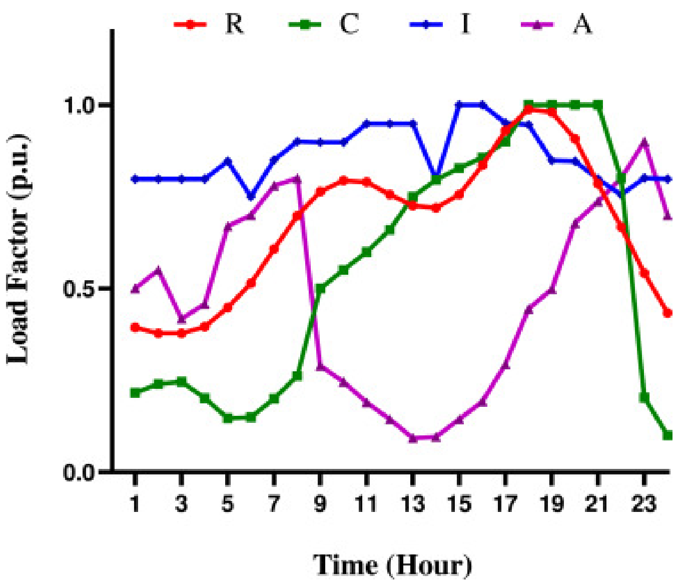

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Fig. 3. Load pattern for different customer classes.

3. Load profile and dynamic pricing

For the assessment of DR, several customer classes i.e. residential, commercial, industrial and agricultural classes have been considered [24]. Each customer class have its unique load pattern and activity usage which is depicted in Fig. 3 in the form of load factor for each class. This gives class-wise load pattern at each node so an aggregated load modelling for each customer class can be performed. The study of these load patterns along with dynamic price is necessary to understand the behaviour of each class after implementation of DR. Fig. 4 specifies the pricing scheme offered by utility to all customer classes and is taken from Ontario Energy Board [38]. The ratio between customer's dynamic price and wholesale market price is taken as 1.45 [39]. This pricing scheme is offered by wholesale market and the overall nature of RTP offered to various customer classes will remain similar to wholesale market price curve though there may be variation in corresponding prices for each customer class due to the subsidy and charging coefficient. These subsidy/charging coefficient are taken into consideration to accommodate government aids for specific customer class and tariff plans for high consuming class. This also serves as a means of maintaining social equity among customer classes by means of cross-subsidy as some customers are overcharged in compensation with some subsidised customers [38]. For each customer class, variable RTP offered on the basis of wholesale market price and subsidy/charging coefficient is given by [40]

$$P^k(t) = P(t)\left[1 + \rho^k\right] \tag{38}$$

Here ρ^k represents subsidy/charging coefficient, which may be different for various customer classes. Eq. (38) forms the basis of dynamic RTP offered to customers which will entice customers to be an active participant in DR scheme. In this pricing scheme, a day of 24 h has been divided into three subsections namely: off-peak hours (00:00 to 07:00 & 23:00 to 24:00), peak hours (08:00 to 11:00 & 18:00 to 22:00) and valley hours (12:00 to 17:00).

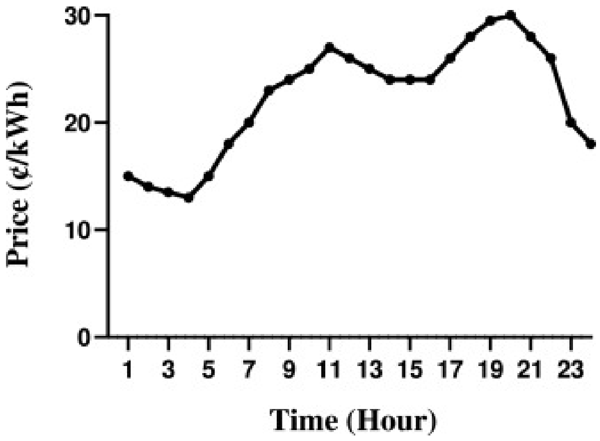

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Fig. 4. RTP offered by utility/ wholesale market.

4. Results and numerical studies

In this section, proposed analytical and stochastic PEM based models have been assessed on IEEE-33 distribution load bus. The system's peak load demand is 3.715 MW and Fig. 5 represents the single line diagram of IEEE-33 distribution load bus [41]. The simulations have been done on MATLAB platform on windows 10 based personal laptop Intel(R) Core(TM) i5 CPU @2.53 GHz, 6 GB RAM. For the analysis purpose, distribution network is considered to be composed of four different type of customer classes: residential (R) customers, commercial (C) customers, industrial (I) customers and agricultural (A) customers respectively as shown in Fig. 5. It is considered that each class's lowest and highest demand range is a portion of their real demand capacity. Table 1 contains class-wise details of demand, allocated nodes, subsidy/charging coefficient and minimum & maximum demand.

The mathematical modelling of elasticity based upon APEM and SPEM have been carried out for different customer classes and results were compared to those of obtained from PEM and base case. The value of self-elasticity is chosen to be −0.3, −0.3, −0.43 & −0.23 for R, C, I & A classes respectively and cross-elasticity is taken as 0.033 [12]. In APEM modelling, the values of elasticity for other than peak hour is calculated based upon Eqs. (24)–(26) where sensitivity coefficient, γ_i^k is assumed to be 0.15. It indicates that maximum demand reduction at the point of peak hour cannot be greater than 15 percent of the corresponding demand [9]. Notably this method eliminates the need of individual self and cross-elasticity on hourly basis and provides a simpler approach. In SPEM method using O-U process, the self-elasticity is taken as same value as that of PEM method whereas the value of cross-elasticity on hourly basis is calculated using Eq. (36). Here, the initial value of cross-elasticity is taken as 0.12 times of self-elasticity of the same class i.e. $\xi(t,t') = |\xi(t,t) \times 0.12|$. In this formulation, the mean and initial value of cross-elasticity is chosen equal to each other. The value of other parameters such as σ and θ are chosen as 1.0 and 0.01 respectively [42]. It is important to mention here that sensitivity coefficient γ^k indicates maximum allowed demand reduction at any point of time. In Equation no. 24, it is being used at the instant of peak price so it is assumed that maximum reduction in demand will happen at this point only. Though this parameter may vary class-wise but for the sake of convenience and uniformity, it is assumed same and capping has been set at 15 percent for all customer classes [9]. While talking about cross-elasticity, it is taken as 0.033 for all customer classes only for PEM case [12]. For APEM and SPEM approaches, cross-elasticity is not kept equal to 0.033 but is calculated using Equation no. 26 and 36 respectively. This way both APEM and SPEM will have distinctive cross-elasticities for each customer class.

Table 1. Customer's class-wise details.

Class	Allocated demand (in %)	Allocated demand (in kW)	Allocated nodes	Subsidy/charging coefficient	Demand range [min, max] (in kW)
R	25.5	950	2 to 10	−0.1	[2, 10]
C	15	555	11 to 18	0.2	[10, 25]
I	34.7	1290	19 to 25	0.5	[50, 100]
A	24.7	920	26 to 33	−0.2	[10, 20]

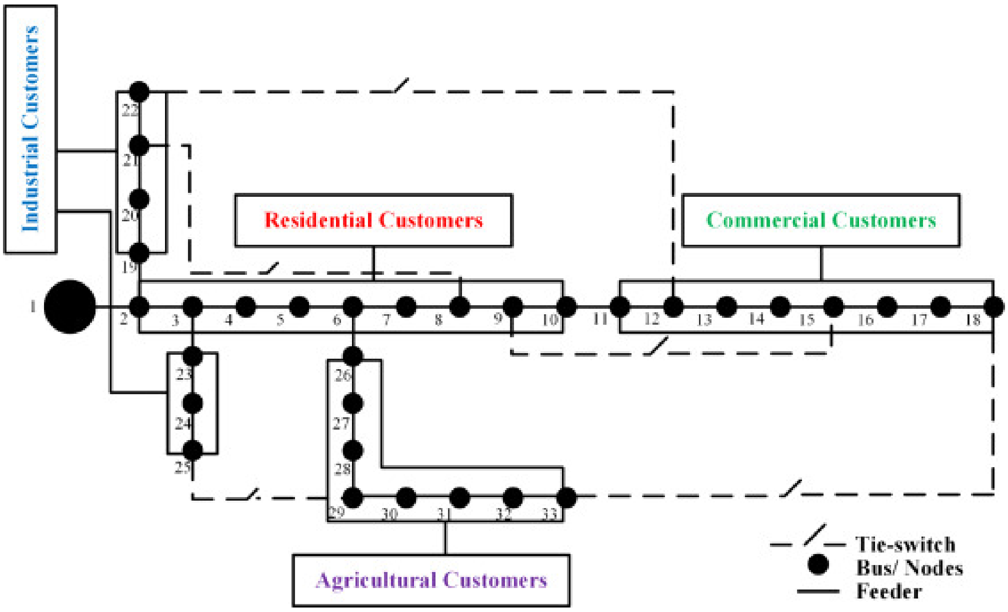

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Fig. 5. Single line diagram of IEEE 33-bus distribution system.

4.1. Class-wise demand after DR

It is shown in Fig. 6(a)–(d) that all customer classes have participated actively in DR program. Here base demand represents BDR demand and PEM, APEM, SPEM & Geometric Brownian motion (GBM) [24] based demands represents ADR demands for each customer class. It is evident from the subfigures that for PEM case, all customer classes have responded in desired fashion and there is reduction in load demand during peak hours and increment in demand during off-peak hours. Though this variation in load demand for each customer class is purely dependent upon chosen values of price elasticity of demand, yet this method is still reliable due to vast applicability of elasticity values. In APEM modelling, cross-peak hour values of elasticity is chosen on the basis of peak hour elasticity and formulation is done maintaining energy balance criterion so that proper load recovery can be observed. A demand-analogy is perceived between peak and off-peak hours and total change in demand during peak and off-peak hours compensates each other. In SPEM modelling, the value of cross-elasticity is randomised using O-U process and suitable results were obtained during simulations. Though, this method exhibits arbitrary nature of customers, still results obtained were comparable to that of base case and other PEM approaches. Both proposed SPEM and GBM approaches are stochastic in nature, yet SPEM display better load shifting behaviour from peak to non-peak hours as compared to GBM. It can be attributed to the presence of variable drift parameter in proposed SPEM in contrast with constant drift associated with GBM.

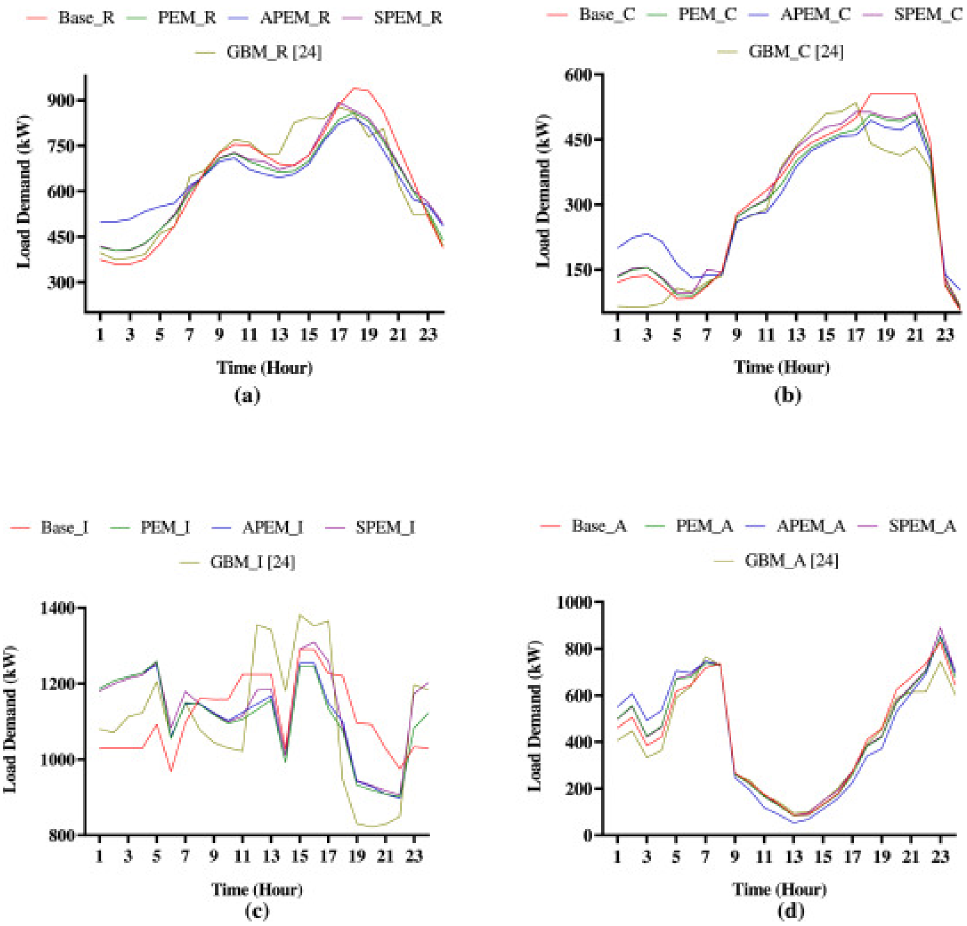

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Fig. 6. Class-wise load demand BDR & ADR for various customer classes: (a) residential customers, (b) commercial customers, (c) industrial customers, (d) agricultural customers.

4.2. Utility load factor after DR

Fig. 7 demonstrates the effect of DR on utility load factor for PEM, APEM, SPEM and GBM [24] approaches. Here utility load factor indicates the pattern of total load demand for all four customer classes. It is shown that utility load factor is improved (increased) considerably for all approaches during off-peak/valley hour whereas it has reduced during peak hours. PEM and SPEM based approaches shows similar kind of response during complete time horizon whereas APEM based approach shows better load factor during off-peak period (0:00 to 07:00) representing maximum increase in demand and marginally low load factor during rest of the period. SPEM method, which is based upon arbitrary nature of customers shows proportionate and more realistic load factor than PEM and APEM due to the applicability of inter-temporal constraints. It can also be seen that the load factor associated with GBM shows very random behaviour, its peak and valley point occur at 17th and 4th hour respectively and there is huge margin between these two points. For the proposed APEM & SPEM approaches, the overall nature of load factor has stabilised and no large peak or dip is observed in any load factor other than that of base case. Moreover peak-to-valley ratio has improved (decreased) considerably following the implementation of the DR programme, which justifies the proposed strategy.

Table 2. Class-wise customers bill for PEM and proposed APEM for segmented time frames.

Class	BDR (¢)			ADR-PEM (¢)			ADR-APEM (¢)		
	Peak	Off-peak	Valley	Peak	Off-peak	Valley	Peak	Off-peak	Valley
R	140844.72	78136.74	90204.78	157766.04	62899.62	96595.18	153199.96	70897.82	94706.85
C	124792.86	31681.62	89076.66	141353.06	24724.32	95247.39	135458.98	36200.66	92799.93
I	384062.64	354526.64	276499.43	415632.87	288686.53	291326.16	418874.1	288545.99	293809.21
A	77006.83	93315.71	15848.54	87310.21	74607.45	17175.51	81657.45	78466.29	14057.29

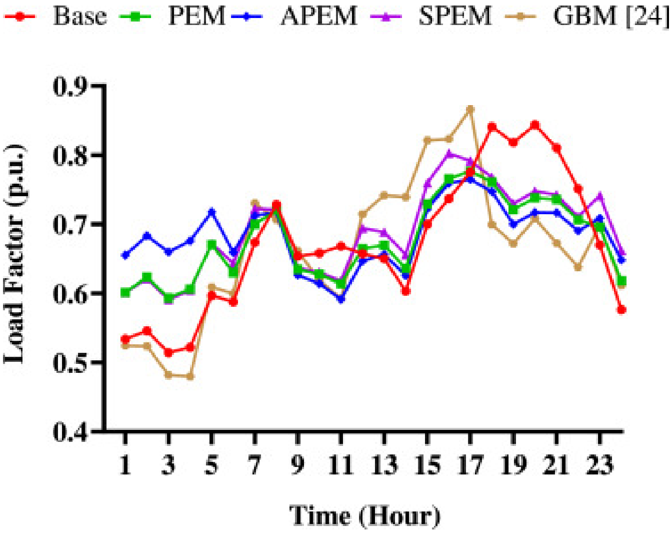

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Fig. 7. Utility load factor for different approaches (a) PEM, (b) APEM, (c) SPEM, (d) GBM [24].

4.3. Class-wise participation in DR

In order to assess class-wise participation, an index term, Demand response participation index (DRPI) is proposed which is defined as the change in aggregated sum of load demand BDR and ADR for peak, off-peak and valley period as a ratio of aggregated sum of demand BDR. DRPI is an indication of how much participation is coming from each class for different time frames as a percentage of total demand. It is defined as

$$DRPI = \left| \frac{\sum_{t \in T_{1/2/3}} D^k(t) - \sum_{t \in T_{1/2/3}} D_o^k(t)}{\sum_{t \in T} D_o(t)} \right| \times 100\% \tag{39}$$

Fig. 8 illustrates several DRPI indexes for each customer class for different time frames for PEM, APEM, SPEM and GBM [24] methods. For PEM, Fig. 8(a) indicates that for R and C classes demand reduction during peak period is not met completely by demand increase during off-peak/valley hours whereas for I and A classes demand increase during off-peak/valley hours overcompensates demand reduction during peak hours. It is due to the reason that load pattern of R and C classes have their most of the peak demand requirements at times of peak dynamic price as opposite to I and A classes. For APEM, Fig. 8(b) indicates that total demand reduction during peak period is completely met by demand increase during off-peak and valley period. For SPEM, Fig. 8(c) indicates approximately same kind of behaviour for C, I and A classes as observed during PEM method. This formulation is dependent upon arbitrary nature of customers so corresponding changes are visible. Fig. 8(d) indicates DRPI for GBM approach and this data has been calculated based upon [24]. Fig. 8(e) summarises aggregated DRPI index for all PEM models and it is evident from this subfigure that for PEM and SPEM methods, demand increase overcompensates demand reduction in contrast to APEM method where both increment and decrement in demand equate each other. Though, GBM is also showing similar kind of participation but at the cost of very random changes in class-wise demand pattern.

Table 3. Class-wise customers bill for PEM and proposed APEM.

Class	**R**	**C**	**I**	**A**	**Overall**
BDR (¢)	309 186.24	245 551.14	1 015 088.46	186 171.08	1 755 996.93
ADR-PEM (¢)	317 260.86	261 324.78	995 645.57	179 093.18	1 753 324.4
Difference (¢)	80 740.62	15 773.63	−19 442.88	−7077.9	−2672.52
% change	2.61	6.42	−1.91	−3.8	−0.15
ADR-APEM (¢)	318 806.64	264 459.58	1 001 229.31	174 181.04	1 758 676.59
Difference (¢)	9620.4	18 908.43	−13 859.14	−11 990.04	2679.65
% change	3.11	7.7	−1.36	−6.44	0.15

Table 4. Class-wise utility profit for PEM and proposed APEM.

Class	**R**	**C**	**I**	**A**	**Overall**
BDR (¢)	57 291.44	119 071.62	599 285.43	30 520.41	806 168.91
ADR-PEM (¢)	65 366.07	134 845.25	579 842.54	23 442.51	803 496.38
Difference (¢)	8074.63	15 773.63	−19 442.89	−7077.9	−2672.52
% change	14.09	13.24	−3.24	−23.19	−0.33
ADR-APEM (¢)	66 911.85	137 980.06	585 426.28	18 530.36	808 848.57
Difference (¢)	9620.41	18 908.44	−13 859.15	−11 990.05	2679.66
% change	16.79	15.87	−2.31	−39.28	0.33

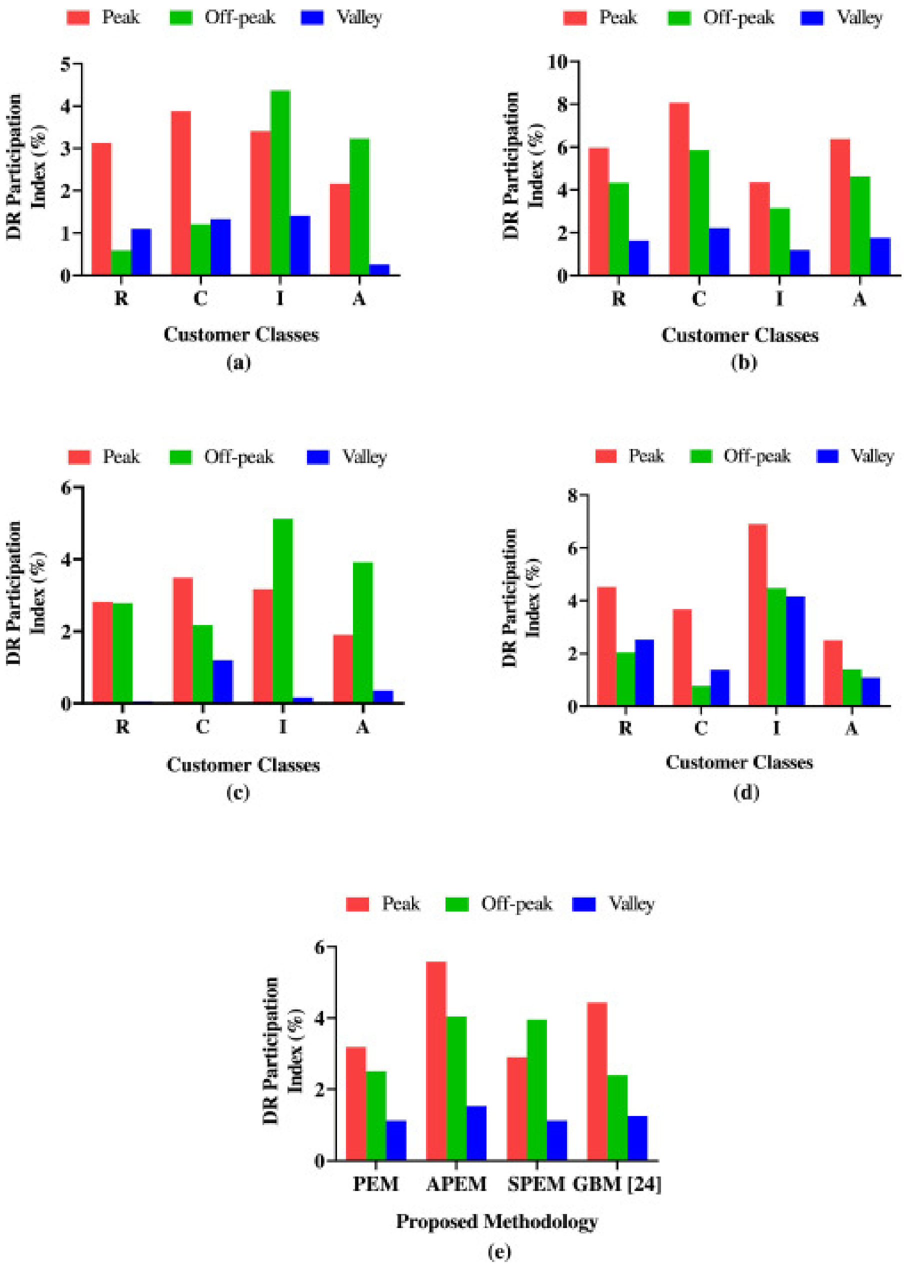

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Fig. 8. DRPI index for (a) PEM, (b) APEM, (c) SPEM, (d) GBM [24] and (e) aggregated DRPI.

4.4. Economic evaluation and performance analysis

In order to assess economic performance, a comparison is done in Table 2, where class-wise billing for segmented time frames is shown for PEM and proposed APEM method. It is shown that for peak hours, bill is increased whereas for off-peak/valley hours, bill is reduced even after participation in DR. In Table 3, an aggregated comparison is done between BDR and ADR methods and it is shown that for the proposed APEM, customer's bill is increased for R and C class whereas for I and A class, customer's bill is reduced. Overall increase in bill is only 0.15% which is not a point of concern so it is evident that customer's bill may or may not get reduced after DR, actually it depends upon load demand pattern and dynamic pricing scheme. SPEM based data are not included in these results due to uncertainty in behavioural pattern, although SPEM based results for flexible, anticipating and postponing type of customers are included in Table 5. In terms of utility profit, Table 4 gives detailed analysis of profit incurred based on margin settled for utility as mentioned in Section 3. Profit has been calculated after subtracting utility cost for producing energy from total revenue earned from customers. Results indicates that significant positive variation is achieved in R and C class whereas huge reduction in utility profit is obtained for A class but the overall change in utility profit is only 0.33%. This variation in economic performance can be attributed to the load pattern of individual class. The proposed methodology may be showing variation in class-wise economic credentials but overall change in customer bill and utility profit do not vary much. Further analysis can be done after extending this work for longer time horizons.

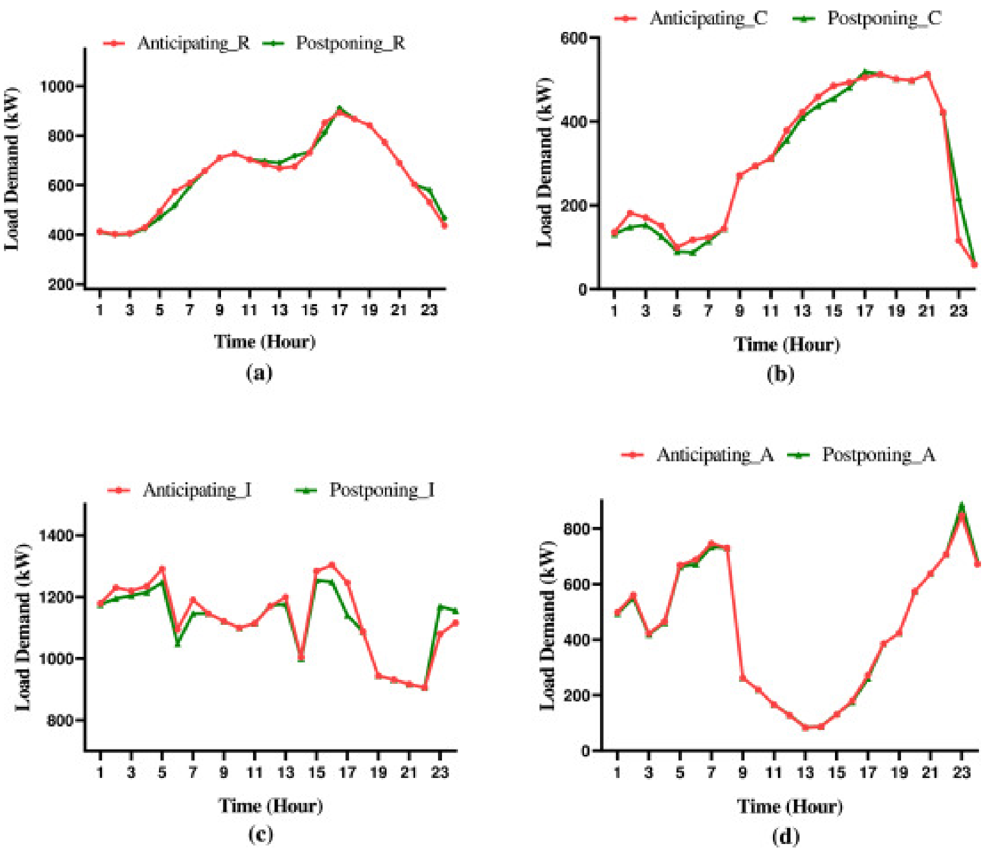

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Fig. 9. Demand plots considering different behavioural patterns of various customer classes: (a) residential, (b) commercial, (c) industrial, (d) agricultural customers.

4.5. Analysis of different behavioural pattern of customers

In this work, SPEM approach has been used for mathematical modelling of different behavioural pattern of customers based on price elasticity of demand. For this task we have assumed three basic behaviour type i.e. flexible customers, anticipating customers and postponing customers. The objective of this study is to show variation in load demand pattern between anticipating type and postponing type of customers as flexible customers are free to change their demand before and after the operating hours. Fig. 9 shows the variation in consumption pattern for various customer classes over complete time horizon and results obtained authenticate the core idea behind anticipating and postponing behaviour.

Fig. 10(a) shows the comparative analysis of demand consumed during first off-peak (00:00 to 07:00) hours and results prove that demand consumed by anticipating customers is more than postponing customers. Similarly in Fig. 10(b) demand pattern is shown for second off-peak (23:00 to 24:00) hours and results prove that demand consumed by postponing customers is greater than anticipating customers. Here in this analysis, the time duration between two peak hours (08:00 to 11:00 and 18:00 to 22:00) i.e. valley hours (12:00 to 17:00) is not considered as this overlapping time zone is before-and-after zone for postponing and anticipating customers respectively so the total demand during this period has mixed response from anticipating and postponing customers that is why it cannot be properly evaluated and compared for any customer type. Table 5 gives detailed analysis of customer's bill, utility profit and energy consumed for flexible, anticipating and postponing customers and results prove that there are very marginal changes observed during any of the behaviour pattern. As the basic approach behind these behaviour pattern in same (SPEM), we can expect some random changes in the data obtained yet all the results lie within limits of 5%. This proves the applicability of the proposed approach. Moreover, economical data obtained for proposed SPEM approach is also compared with another stochastic method, Geometric Brownian motion (GBM) [24] which is considered for flexible customers only. Table 5 (upper half) provides a comprehensive assessment of customer bill, utility profit and energy consumed for proposed O-U process and GBM method.

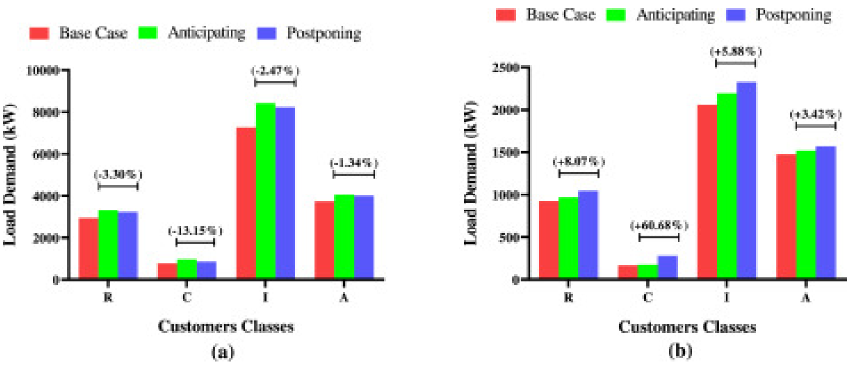

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Fig. 10. Class-wise consumed demand for different behavioural pattern during (a) 00:00 to 07:00 and (b) 23:00 to 24:00 h.

Table 5. Comparative assessment for different behavioural patterns.

IEEE system	Method used	Customer's bill (¢)	Difference (% change)	Utility profit (¢)	difference (% change)	Energy consumed (kWh)	% change
Standard 33-bus distribution system	Base	1755996.93	–	806168.91	–	59864.32	–
	SPEM (flexible)	1797206.91	41209 (2.34)	847378.89	41209 (5.11)	60568.01	1.17
	SPEM (anticipating)	1791593.43	35596.5 (2.02)	850959.29	44790.38 (5.55)	60407.69	0.907
	SPEM (postponing)	1778881.19	22884.93 (1.3)	843262.58	37093.67 (4.6)	60016.77	0.25

IEEE system	Method used	Customer's bill (¢)	Difference (% change)	Utility profit (¢)	difference (% change)	Energy consumed (kWh)	% change
	GBM [24]	1793152.6	37156 (2.11)	840715.89	34547 (4.28)	60492	1.04
Modified 33-bus distribution system [43]	SPEM (flexible)	1797015.81	41018 (2.33)	847065.25	40896 (5.07)	60607.81	1.24
	SPEM (anticipating)	1791664.28	35667.35 (2.03)	851083.31	44914.41 (5.57)	60394.2	0.885
	SPEM (postponing)	1780075.93	24079 (1.37)	842998.86	36829.96 (4.56)	59917.18	0.08
	GBM [24]	1792604	36607 (2.084)	841018.19	34849.28 (4.32)	60460	0.99

4.6. Modified IEEE 33-bus distribution system (DS)

The proposed SPEM and the existing GBM approach is also investigated on the modified IEEE 33-bus distribution system [43]. The considered system is modelled similar to the standard 33-bus system and same RTP pricing is employed. The considered modified 33-bus system is different from standard system in terms of enhancement of generation capacity through DGs, reactive power compensation and min/max voltage constraints. As there is no change proposed in load demand profile and corresponding line data, it is observed that the class-wise load pattern and associated load factor for PEM and APEM approach remains unaffected when tested on modified system, although marginal changes are observed for SPEM approach and that too are due to the randomness associated with different behavioural customers. Table 5 (lower half) presents similar economical comparison for different customers for modified IEEE 33-bus distribution system as presented with standard bus system. Moreover, the active power losses associated with different behavioural customers on different distribution systems vary in significant proportion. The percentage contribution towards total active power losses for peak and off-peak/valley hours is given as

$$\%P_{loss_{,t\in T_{1/2/3}}} = \frac{P_{loss_{,t\in T_{1/2/3}}}}{P_{loss_{,t\in T}}} \times 100 \tag{40}$$

The slot-wise variation in percentage contribution towards total active power losses for BDR and ADR load profile is given by

$$\%\Delta P_{loss_{,t\in T_{1/2/3}}} = \%P^{ADR}{}_{loss_{,t\in T_{1/2/3}}} - \%P^{BDR}{}_{loss_{,t\in T_{1/2/3}}} \tag{41}$$

Table 6 presents an extensive slot-wise comparison of percentage contribution in total active power losses before and after DR for different behavioural customers on both standard and modified DS. It is observed that for both IEEE systems and for all three types of customers, active power losses get reduced during peak hours and gets reflected back during off-peak/valley hours. Table 7 represents slot-wise variation in percentage contribution between standard and modified IEEE system for different type of customers. It can be inferred that both distribution systems transfer power losses fairly from peak to non-peak hours and slot-wise variation in percentage contribution towards total active power losses is more or less same for all three considered behavioural customers.

Table 6. Slot-wise contribution (in %) in total active power losses for standard and modified IEEE 33-bus systems.

Time-slot	Standard 33-bus DS				Modified 33-bus DS			
	BDR (%)	ADR (%)			BDR (%)	ADR (%)		
		Flexible	Anticipating	Postponing		Flexible	Anticipating	Postponing
Peak	43.00	39.92	40.02	40.22	41.54	39.26	39.36	39.47
Off-peak	32.29	35.11	34.98	35.12	33.84	35.94	35.86	36.00
Valley	24.69	24.95	24.98	24.64	24.60	24.79	24.76	24.52

Table 7. Slot-wise variation in contribution (in %) towards total active power losses.

Time-slot	Flexible		Anticipating		Postponing	
	Standard 33-bus DS	Modified 33-bus DS	Standard 33-bus DS	Modified 33-bus DS	Standard 33-bus DS	Modified 33-bus DS
Peak	−3.07	−2.97	−2.77	−2.28	−2.18	−2.07
Off-peak	2.81	2.69	2.83	2.09	2.01	2.15
Valley	0.25	0.28	−0.05	0.18	0.16	−0.07

4.7. Quantitative analysis and discussion of results

In this section, firstly the distinction between basic and proposed modelling approaches is discussed which is examined for individual class-wise demand pattern under the ambit of dynamic pricing. Further, the impact of proposed approaches is analysed on load factor, customer bills and utility profit. Moreover, the variation in demand consumption between anticipating and postponing customers is analysed for complete time horizon and off-peak hours. These findings are summarised below:

1. On comparing numerical values obtained using Fig. 6 as mentioned in Section 4.1, it is seen that peak curtailment during peak hours is maximum for commercial customers (15% and 10.2% for APEM and SPEM), lowest for agricultural customers (6% and 3.8% for APEM and SPEM) and moderate for both residential (10.2% and 7.6% for APEM and SPEM) and industrial (8.2% and 9% for APEM and SPEM) customers.

2. From the load factor (LF) perspective as shown in Fig. 7 in Section 4.2, both APEM and SPEM approaches have shown substantial increment and decrement in its values during off-peak and peak hours respectively. However maximum improvement in LF

with respect to before-DR values is seen during 4th hour where both APEM and SPEM approaches provides 29.5% and 15.7% increment respectively. Similarly, both APEM and SPEM approaches provides maximum decrement of 15% and 11.3% during 20th hour respectively. It is due to the fact that at 4th and 20th hour, dynamic price is at its lowest and highest value respectively so maximum changes are being observed at these instants. Moreover, DR benefits LF in terms of peak-to-valley (P2V) distance as well, APEM improves (reduces) P2V by 47.33% whereas SPEM improves P2V by 35.7%.

3. From the customers point of view, Table 2 shows the detailed analysis of class-wise customer bill segmented for different time frames. This analysis has been extended in Table 3, where a proper comparison between BDR customer bill and ADR customer bill comprising PEM and APEM approach is presented. It can be seen that for APEM approach, the overall increment in bill is merely 0.15%, though we assume that customer bill should get reduced after participating in DR, yet the net ADR bill depends upon several things such as updated demand, RTP and government induced subsidy. In terms of utility profit, it can be seen from Table 4 that overall change in utility profit is 0.33% as compared to BDR profit. Class-wise profit is a result of updated demand and the ratio between customer's dynamic price and wholesale market price. It can be asserted from this inference that even after participating in DR, some customers may not get benefit as policymakers are mainly responsible for proper functioning of electricity grid.

4. While considering SPEM for modelling of different behavioural patterns, Fig. 9, Fig. 10 presents a clear picture of demand consumption (in kW) and demand variation (in %) for anticipating and postponing customers for complete time horizon and for off-peak hours. Table 5 presents a thorough examination of customer bill, utility profit and energy consumed for flexible, anticipating and postponing customers for both standard and modified IEEE 33-bus distribution systems. It can be inferred that from customer point of view, postponing approach seems to be most beneficial as overall bill and energy consumed is lowest in this approach whereas from utility point of view, flexible and anticipating approaches are preferable. Though percentage variation among the results of these approaches do not vary much, it still provides customers an estimation of probable economic data after participation in DR.

5. The proposed APEM and SPEM methodologies are suitable for both PBDR and IBDR as these methodologies are used for elasticity modelling and are dependent upon transformation of pricing structure from flat to dynamic one; however this paper has derived all equations, results and discussions only for PBDR and does not explore IBDR due to non-inclusion of incentive/penalty values.

4.8. Limitations of the work

The proposed work has some limitations which are mentioned here:

1. In this work subsidy/charging coefficient ρ^k (as specified in Table 1) has been assumed constant for individual customer class which represent constant change in

dynamic price for all customers of specified class, though some of the customers may not need it (big industrial players), whereas some may require it more (small agricultural and residential customers).

2. The Sensitivity coefficient γ^k, which indicates maximum allowed demand reduction at any point of time is assumed same for all customer classes.

3. For PEM formulation, the value of cross-subsidy has been assumed equal to 0.033, which is kept same and constant for all customer classes and for complete time horizon. Although these values are different and variable for APEM and SPEM approach.

4. The main limitation and disadvantage of PEM method is the inclusion of same and constant value of cross-elasticity for complete time horizon and for all classes, due to which exact replication of customer behaviour in response to dynamic pricing may not be visible in load pattern.

5. Conclusions

In this paper, a mathematical model based upon price elasticity of demand is presented to accommodate the features of DR in distribution system. The proposed work basis its operation on dynamic elasticity using PEM and this dynamic elasticity is modelled using analytical and stochastic approach. Both the approaches have been used for modelling of cross-elasticity for PEM of various customer classes. Analytical approach is dependent upon load recovery and energy balance principle whereas stochastic modelling has been presented using Ornstein–Uhlenbeck process to imitate arbitrary nature of customers. The process is a variation of the cross-elasticity in continuous time, in which the features of the process have been adjusted so that the cross-elasticity has a stronger propensity to return to its central/mean position whenever the process is moving away. Stochastic modelling using Ornstein–Uhlenbeck process have been further extended for anticipating and postponing type of customers to get some insight of their demand pattern. The incorporation of both analytical and stochastic modelling approach enhances the robustness of this work. Both the proposed approaches are beneficial in nature and easy to implement. APEM calculates elasticity values on the basis of peak demand which vary class-wise thus eliminates Simpsons paradox in results whereas SPEM provides an easy formulation inculcating the stochasticity of human behaviour in demand response. The utilisation of the Ornstein–Uhlenbeck process in capturing the stochastic nature of customers makes it more realistic reflection of consumer behaviour. Overall results prove that technically and economically, benefit of DR implementation is not universal, though it may be beneficial for particular classes. Load factor improves its shape significantly as it gets comparatively flattened and sudden peak and dip do not appear in it. This conclude that a common pricing scheme may or may not be beneficial for all customer classes due to the diverse nature of their consumption pattern and there is a need of class-wise pricing scheme.

CRediT authorship contribution statement

Gaurav Kansal: Conception and design of study, Acquisition of data, Analysis and/or interpretation of data, Writing – original draft. **Rajive Tiwari:** Analysis and/or interpretation of data, Writing – review

& editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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All authors approved the version of the manuscript to be published.

Recommended articles

Data availability

No data was used for the research described in the article.

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