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CHAOTIC INFLATION IN BIANCHI TYPE V SPACE-TIME WITH CONSTANT DECELERATION PARAMETER AND BAROTROPIC PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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Abstract: Exact cosmological solutions of the Einstein gravitational equations are obtained for chaotic inflationary cosmological model in Bianchi V space-time in barotropic perfect fluid distribution. Inflationary scenario in Bianchi Type V space-time with variable bulk viscosity is discussed. To get deterministic solution we consider deceleration parameter as constant which shows acceleration and decelerating universe as constant value is greater or less than 0 respectively. The special volume increases with time and this expansion continuous for long enough which is inflationary scenario in universe. The model have initially point type singularity.

Key words: Chaotic, cosmological model, Bianchi, perfect fluid, deceleration.

1. Introduction

On the basis of available observation it was believed that universe was created in hot big-bang. At the time of Big Bang most of the energy was in the form of radiation. Astronomical observations in late eighties revealed that predictions of FRW models do not always meet our requirements as was believed earlier Smoot et al. [20]. Therefore specially homogeneous and anisotropic Bianchi space time (I-IX) is considered to study the universe in its early stage of evolution. Among these Bianchi type V model create more interest in the study because these models contain isotropic special cases and allow small anisotropy levels.

At any instant of cosmic time, Bianchi type V space time is the natural generalization of FRW model with negative curvature. These models have been studied by number of authors viz. Collins [10], Ram [19], Banerjee and Sanyal [7], Bali and Kumawat [5], Bali et al. [6]

Guth [12] introduced the concept of inflationary phase. Inflation is the rapid exponential expansion of the early universe by a factor of 10^{78} in volume driven by a negative vacuum energy density. The inflationary universe incorporates all the predictions of the model for the universe because inflationary Universe has the same behavior after 10^{-32} seconds. Guth has discussed the inflationary universe as a possible natural expansion for the observed large scale homogeneity and near critical density of the universe expansion. Later on Anninous et al. [1] discussed the significance of inflation for isotropization of universe. Stein-Schabes [21] has explained that inflation will take place if effective potential has flat region where Higgs field evolves slowly but the universe expands in an exponential way. Burd [9] discussed inflationary scenario in FRW model. The inflationary universe is investigated by number of authors viz. Linde [13], Wald [22], Barrow [8], Ellis and Madsen [11], Bali and Jain [4], Bali [2] within the framework of model which is already homogeneous and isotropic.

The inflationary Universe scenario provides a satisfactory solution to some of the conceptual issues in cosmology not understood in the standard Big Bang Theory. Maartens et al. [16] studied chaotic inflation on the brain using a massive scalar field. Chaotic inflation on the FRW brane was obtained with an assumption that the potential energy dominates over that of the kinetic energy of the scalar field. Linde [14] has shown that chaotic scenario can be realized even when scalar field ø obeys $\frac{1}{2}\dot{\varphi}^2 \gg V(\varphi)$ and attains the required condition for inflation in course of its evolution in FRW universe. Paul et al. [18] have pointed out that Linde's chaotic inflationary scenario is fairly general and can be accommodated even if the universe is anisotropic. Bali [3] investigated chaotic inflationary Universe scenario assuming $R^3 = e^{3H_0 t}$ and following Linde [15] assuming $\ddot{\varphi} \ll \frac{dV}{d\varphi}$.

In this paper we have investigated chaotic inflationary scenario in Bianchi type V space time assuming $V(\varphi) = \lambda\varphi^n, \lambda$ being constant.

The deceleration in cosmology is a dimensionless measure of the cosmic acceleration of the expansion of space in FRW universe. The expansion of the universe is said to be accelerating if q < 0 and decelerating if q > 0. Thus it is interesting to assume that decelerating parameter is constant with the help of which we can explain the decelerating and accelerating phase of the universe.

To get deterministic model we also assume the deceleration parameter is constant.

2. Metric and Field Equations

We consider Bianchi Type V line-element in orthogonal form as

$$ds^2 = -dt^2 + A^2\, dx^2 + e^{2x}\,(B^2 dy^2 + C^2\, dz^2) \tag{1}$$

where A, B, C are metric potentials and are function of t-alone.

We assume the coordinates to be comoving so that $v^1 = v^2 = v^3 = 0, v^4 = 1.$

The Lagrangian in which gravity is minimally coupled to the scalar field (ϕ) is given as

$$L = \int \sqrt{-g}\left[R - \frac{1}{2} g^{ij} \partial_i \phi \partial_j \phi - V(\phi)\right] d^4 x \tag{2}$$

Modified Einstein's field equation (in gravitational units 8πG = c = 1) given as

$$R_{ij} - \frac{1}{2} R\, g_{ij} = - T_{ij} \tag{3}$$

The cosmic fluid is assumed to be perfect fluid.

The energy momentum tensor with scalar field (ϕ) for perfect fluid distribution is given as

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} + \partial_i \phi \partial_j \phi - \left\{\frac{1}{2} \partial_k \phi \partial^k \phi + V(\phi)\right\} g_{ij} \tag{4}$$

where ρ is the matter density, p the isotropic pressure and V(ϕ) is the potential.

The energy conservation law coincides with the equation of motion for ϕ and we have

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g}\, \partial^\mu \phi) = - \frac{dV}{d\phi} \tag{5}$$

This leads to

$$\phi_{44} + \phi_4 \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{dV(\phi)}{d\phi} \tag{6}$$

The homogeneous scalar field ϕ which is identified with the inflation is only function of cosmic time t and $\phi_4 = \frac{d\phi}{dt}$.

The Einstein's field equation (3) for the space-time (1) with equation (4) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{1}{A^2} = -\left[p + \frac{1}{2}\phi_4^2 - V(\phi)\right] \tag{7}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{1}{A^2} = -\left[p + \frac{1}{2}\phi_4^2 - V(\phi)\right] \tag{8}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{1}{A^2}=-\left[p+\frac{1}{2}\phi_4^2-V(\phi)\right] \tag{9}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{3}{A^2}=-\left[-\rho-\frac{1}{2}\phi_4^2-V(\phi)\right] \tag{10}$$

$$\frac{2A_4}{A}-\frac{B_4}{B}-\frac{C_4}{C}=0 \tag{11}$$

Equation (11) leads to

$$A^2=kBC$$

where k is constant of integration.

For simplicity we assume constant of integration unity i.e.

$$A^2=BC \tag{12}$$

3. Solution of Field Equations

Equation (8) and (9) leads to

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{A_4}{A}\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=0$$

$$\frac{B_{44}}{B}-\frac{C_{44}}{C}+\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)\left(\frac{B_4}{B}-\frac{C_4}{C}\right)=0$$

$$\frac{(CB_4-BC_4)_4}{CB_4-BC_4}=-\frac{1}{2}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)$$

$$C^2\left(\frac{B}{C}\right)_4=\frac{L}{(BC)^{1/2}} \tag{13}$$

Let B/C= ν and BC = μ above equation leads to

$$\frac{\nu_4}{\nu}=\frac{L}{\mu^{3/2}} \tag{14}$$

To find deterministic model let

(i) Deceleration parameter q is constant.

(ii) $V(\emptyset)=\lambda\phi^n$

(iii) $\rho = 3H^2$

(iv) $p=\gamma\rho \quad ,0 \leq \gamma \leq 1$

Deceleration parameter in cosmology is a dimensionless measure of the cosmic acceleration of the expansion of space in FRW universe. It is defined as

$$q = -\frac{R_{44}/R}{R_4^2/R^2} = \alpha(costant) \tag{15}$$

Where R is scale factor of the universe.

Equation (15) leads to

$$\frac{R_{44}}{R} + \alpha\left(\frac{R_4}{R}\right) = 0$$

$$R = (at+b)^{\frac{1}{\alpha+1}} \tag{16}$$

Where a=(α+1)β, β is constant of integration

b=(α+1)β', β' is constant of integration

The scale factor R for the line-element (1) is given by

$$R^3 = ABC$$

Using (16) this leads to

$$ABC = (at+b)^{\frac{3}{\alpha+1}}$$

Using (12) this gives

$$\frac{B_4}{B} + \frac{C_4}{C} = \frac{2a}{(\alpha+1)(at+b)}$$

$$\frac{\mu_4}{\mu} = \frac{2a}{(\alpha+1)(at+b)}$$

$$\mu = c(at+b)^{\frac{2}{\alpha+1}} \tag{17}$$

Using this in (14) we get

$$\nu = d\exp\left\{\frac{L(\alpha+1)}{a(\alpha-2)c^{3/2}}(at+b)^{\frac{\alpha-2}{\alpha+1}}\right\} \tag{18}$$

Since $B^2 = \mu\nu, C^2 = \frac{\mu}{\nu}$

$$B = \left[cd\,(at+b)^{2/\alpha+1} \exp\left\{ \frac{L(\alpha+1)}{a(\alpha-2)c^{3/2}} (at+b)^{\frac{\alpha-2}{\alpha+1}} \right\} \right]^{1/2} \tag{19}$$

$$C = \left[\frac{c}{d}(at+b)^{2/\alpha+1} \frac{1}{\exp\left\{ \frac{L(\alpha+1)}{a(\alpha-2)c^{3/2}} (at+b)^{\frac{\alpha-2}{\alpha+1}} \right\}} \right]^{1/2} \tag{20}$$

$$A = \sqrt{c}(at+b)^{\frac{1}{\alpha+1}} \tag{21}$$

Or

$$A = \sqrt{c}T^{\frac{1}{\alpha+1}}$$

$$B = \left[\eta T^{2/\alpha+1} \exp\left\{ lT^{\frac{\alpha-2}{\alpha+1}} \right\} \right]^{1/2}$$

$$C = \left[\kappa T^{2/\alpha+1} \frac{1}{\exp\left\{ lT^{\frac{\alpha-2}{\alpha+1}} \right\}} \right]^{1/2}$$

Where T = (at+b), $l = \frac{L(\alpha+1)}{a(\alpha-2)c^{3/2}}$, $\eta = cd, \kappa = c/d$

Therefore, the metric (1) leads to

$$\mathrm{ds}^2 = -\frac{\mathrm{dT}^2}{\mathrm{a}^2} + cT^{\frac{2}{\alpha+1}}dx^2 + e^{2x}\left[\eta T^{2/\alpha+1} \exp\left\{ lT^{\frac{\alpha-2}{\alpha+1}} \right\} \right]\mathrm{dy}^2 + e^{2x}\left[\kappa T^{2/\alpha+1} \frac{1}{\exp\left\{ lT^{\frac{\alpha-2}{\alpha+1}} \right\}} \right]\mathrm{dz}^2 \tag{23}$$

For Chaotic inflation

Let $V(\emptyset) = \lambda\phi^n$

$$\frac{dV(\phi)}{d\phi} = \lambda n\phi^{n-1} \tag{24}$$

From (6)

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{dV(\phi)}{d\phi}$$

Following Linde [14] for chaotic inflation, we consider

$$\phi_{44} \ll \frac{dV}{d\phi}$$

So from (17)

$$\phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \lambda n\phi^{n-1}$$

$$\Rightarrow \frac{d\phi}{dt}\left[\frac{3a}{(\alpha+1)(at+b)}\right] = \lambda n\phi^{n-1}$$

$$\Rightarrow \frac{d\phi}{\phi^{n-1}} = \frac{\lambda n(\alpha+1)}{3a}(at+b)dt$$

$$\Rightarrow \phi^{2-n} = (2-n)\left[\frac{\lambda n(\alpha+1)}{6a^2}(at+b)^2 + e\right] \tag{25}$$

Where e is constant of integration.

4. Physical and Geometrical Aspects

The expansion (θ), Hubble parameter (H), matter density (ρ), isotropic pressure (p), spatial volume (R^3), shear (σ) for the model (23) are given by

Expansion $\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$

$$= \frac{3a}{(\alpha+1)T} \tag{26}$$

Hubble parameter $H = \frac{\theta}{3} = \frac{a}{(\alpha+1)T}$ (27)

Matter density $\rho = 3H^2 = \frac{3a^2}{(\alpha+1)^2T^2}$ (28)

Isotropic pressure p= $\gamma\rho = \gamma \frac{3a^2}{(\alpha+1)^2 T^2}$, $0 \le \gamma \le 1$ (29)

Spatial volume $R^3 = ABC = T^{3/\alpha+1}$ (30)

Shear $\sigma = \frac{1}{2}\left(\frac{B_4}{B} - \frac{C_4}{C}\right)$

$$= \frac{L}{2C^{3/2} T^{3/\alpha+1}} \tag{31}$$

Now ratio of shear and expansion

$$\frac{\sigma}{\theta} = \frac{L(\alpha+1)}{6aC^{3/2} T^{2-\alpha/\alpha+1}} \tag{32}$$

Thus for large values of T (α < 2), $\frac{\sigma}{\theta} \to 0$

Anisotropic parameter (Â):

Anisotropic parameter Â is defined as

$$\hat{A} = \frac{1}{3}\left[\left(\frac{H_1}{H} - 1\right)^2 - \left(\frac{H_2}{H} - 1\right)^2 - \left(\frac{H_3}{H} - 1\right)^2\right]$$

Where $H_1 = \frac{A_4}{A}$, $H_2 = \frac{B_4}{B}$, $H_3 = \frac{C_4}{C}$

Thus $H = \frac{a}{(\alpha+1)T}$

$$H_1 = \frac{a}{(\alpha+1)T}$$

$$H_2 = \frac{a}{(\alpha+1)T} + \frac{la(\alpha-2)}{2(\alpha+1)} T^{-3/\alpha+1}$$

$$H_3 = \frac{a}{(\alpha+1)T} - \frac{la(\alpha-2)}{2(\alpha+1)} T^{-3/\alpha+1}$$

So $$\hat{A} = \frac{l^2(\alpha-2)^2}{6} T^{\frac{-2(2-\alpha)}{\alpha+1}} \tag{33}$$

Â→0 for large values of T(α < 2)

The slow roll parameters ε and δ is defined by Unnikrishnan and Sahni [23] as

$$\epsilon = -\frac{H_4}{H^2} = \frac{1}{a} \tag{34}$$

and

$$\delta = \varepsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \dot{\epsilon} = \frac{\partial \epsilon}{\partial t} \tag{35}$$

$$\varepsilon = \frac{1}{a} \tag{36}$$

Thus slow role PLI (Power Law Inflation) corresponds to $\varepsilon << 1$ which occurs when $a >> 1$.

5. Conclusion

The spatial volume (R^3) increases with time which represents inflationary scenario in the universe. The rate of expansion slows down with the increase of cosmic time and $\theta \to 0$ as $T \to \infty$.

The Hubble parameter (H) is initially large but decreases with time. The average anisotropic parameter ($\hat{A}$) is not zero initially i.e. anisotropy is initially large but disappears for large values of T therefore model isotropizes at late time. The model have point type singularity at T=0. Since deceleration parameter q is constant therefore the model represents decelerating ($\alpha > 0$) and accelerating ($\alpha < 0$) phase of universe. The scalar field ϕ decreases with time. The model (23) indicates that energy density ρ is large initially and decreases during expansion.

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