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Applications of the Laplace variational iteration method to fractional heat like equations

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Abstract

The importance of differential equations of integer order and fractional order can be seen in many areas of engineering and applied sciences. The present work involves fractional order heat equations that arise in numerous applications of engineering and aims to find series solutions by the Laplace variational iteration method (LVIM). The method combines the Laplace transform and the variational iteration method. To show the efficiency and validity of LVIM, we have exemplarily considered 1-D, 2-D, and 3-D fractional heat equations and solve them by LVIM. Exact solutions are gained in expressions of the Mittag-Leffler function. The results are also explored through graphs and charts.

Keywords

Fractional differential equations; Laplace transform; Variational iteration method; Caputo derivative

1. Introduction

Fractional calculus can be used to present facts and deeper aspects in different fields such as Physics, Chemistry, Engineering, and other fields of applied science and applied mathematics. From a theoretical and practical point of view, the region presents several fundamental problems[1] and the recent developments as well as its applications.[2], [3], [4] Like the parental importance, the fractional differential equations have many applications in different streams of engineering and applied sciences. The significance of these equations can demonstrate non-linear oscillation of earthquake, electromagnetism, electrochemistry, acoustics, signal processing, diffusion processes[5], [6], [7] and many other areas. Some very recent work presented by Mastoi etal.,[8] Ghanbari[9] and Djilali and Ghanbari,[10] Yadav etal.,[11] Ramani etal. [12] can be referred for the latest update in the field.

The heat equation is a significant partial differential equation, which was developed by Joseph Fourier in 1822. These equation expresses the distribution of heat (or variation of temperature) in a section over time. These equations have reputation in various scientific grounds. It is an ideal parabolic partial differential equation in mathematics and related to the 'Brownian motion' via the Fokker–Planck equation.[13,14] The diffusion equation is the generalized version of the heat equation, emerges in relation with the learning of chemical diffusion and other processes respectively. The heat equation tells that if a warm body is put in a container of cold water, the body temperature will reduce, and ultimately (after a particular period, and on a condition of no external heat provided) the temperature will reach the state of equilibrium. Authors like Jafari etal.,[1] Yang and Machado,[15] Mastoi etal.,[8] Rüland and Salo,[16] and Mamun etal. [17] are in the list those have contributed their concerned work in the field.

For the current issue, we take into consideration the following fractional heat equations with variable coefficients:

$$D_{\tau}^{\vartheta} u = F(\alpha,\beta,\gamma)\frac{\partial^2 u}{\partial \alpha^2} + G(\alpha,\beta,\gamma)\frac{\partial^2 u}{\partial \beta^2} + H(\alpha,\beta,\gamma)\frac{\partial^2 u}{\partial \gamma^2} \quad , \quad 0 < \vartheta \leq 1 \tag{1.1}$$

with the initial conditions

$$u(\alpha,\beta,\gamma,0) = h(\alpha,\beta,\gamma), \quad u_t = (\alpha,\beta,\gamma,0) = m(\alpha,\beta,\gamma). \tag{1.2}$$

An analytical approach that is more powerful than the traditional variational technique is named as "Variational iteration method" (VIM), which was initially recommended by He.[18] The "Laplace variational iteration method" (LVIM) is a combination of the "Laplace transform" and "variational iteration method". Applications of VIM to fractional differential equations are slow to converge, mainly because they directly use the Lagrange multipliers of ordinary differential equations (ODEs).[19] Wu and Baleanu[20] pointed out that it can be difficult to apply integrals by parts of the Riemann–Liouville (RL) integral resulting from the constructed correction function. Zada etal. [21] established new iterative approach for the solutions of fractional order inhomogeneous partial differential equations. To overcome this shortcoming, they proposed to identify generalized Lagrange multipliers via the Laplace transform. Without the need for linearization, discretization, or perturbation, the LVIM is a form of semi-analytical methodology that can be used with both linear and non-linear equations. This method is better than Finite Difference Method and the Finite Element Method. It is not a time-consuming procedure and provides an accurate and error-free solution quickly. This method has been utilized by many authors to solve several problems.[22], [23], [24] Applying the LVIM to solve heat equations is what makes this study novel, and its approach has also been found to converge rapidly to the exact solution of the fractional heat-like equations.

This approach of the LVIM demonstrates the effectiveness of the method to get the precise and more accurate solution of the exemplary problems considered here. For the future prospects, it may be assumed that observing the solution of the problems discussed here, the LVIM may be used to solve more sensitive problems of science containing ODEs and Fractional differential equations[25], [26], [27] with more precision.

Preliminaries

Definition 1.1

The Caputo derivative[28] of random order of the function $u(\alpha,\tau)$ is well-defined as

$$D_\tau^\vartheta u(\alpha,\tau)= \frac{1}{\Gamma(m-\vartheta)}\int_0^\tau (\tau-\delta)^{m-\vartheta-1}u^{(m)}(\alpha,\delta)d\delta = J_\tau^{m-\vartheta}D^m u(\alpha,\tau), \quad (1.3)$$

$$(m-1<\vartheta\le m, m\in\mathbb{N}),$$

where $D^\vartheta=\frac{d^\vartheta}{d\tau^\vartheta}$ and $J_\tau^\vartheta(.)$ displays the Riemann–Liouville integral operator of fractional order,[29] $\vartheta>0$.

$$J_\tau^\vartheta u(\alpha,\tau)= \frac{1}{\Gamma(\vartheta)}\int_0^\tau (\tau-\delta)^{\vartheta-1}u(\alpha,\delta)d\delta, \quad (1.4)$$
$$(\delta>0, m-1<\vartheta\le m, m\in\mathbb{N}).$$

Definition 1.2

The Laplace Transform[30] of $f(\tau),\quad \tau>0$ is well-defined as

$$\mathscr{L}[f(\tau)]= F(s)= \int_0^\infty e^{-s\tau}f(\tau)d\tau. \quad (1.5)$$

Definition 1.3

The Laplace transform of $D_t^\vartheta u(x',t)$ is defined as

$$\mathscr{L}\left[D_\tau^\vartheta u(\alpha,\tau)\right]= \mathscr{L}[u(\alpha,\tau)] \quad (1.6)$$
$$-\sum_{n=0}^{m-1} u^n(\alpha,0)s^{\vartheta-n-1},(m-1<\vartheta\le m, m\in\mathbb{N}).$$

Definition 1.4

The Mittag-Leffler function[31] is described as

$$E_\vartheta(\tau)= \sum_{n=0}^\infty \frac{\tau^n}{\Gamma(\vartheta n+1)}, \qquad (\vartheta\in\mathbb{C},\quad \Re(\vartheta)>0). \quad (1.7)$$

$$E_{\vartheta,\theta}(\tau)= \sum_{n=0}^\infty \frac{\tau^n}{\Gamma(\vartheta n+\theta)}, \qquad (\vartheta,\theta\in\mathbb{C},\quad \Re(\vartheta)>0,\Re(\theta)>0).$$

Variational Iteration Method (VIM)

The VIM recognized by He[18] and it has wide applications to analyse either accurate or aggregate solutions of linear and nonlinear difficulties.[32], [33], [34], [35] The VIM gives the solution in an infinite rapidly convergent series. We deliberate the succeeding equation to demonstrate the perception of VIM:

$$\mathrm{L}u\left(\alpha,\tau\right)+\mathrm{N}u\left(\alpha,\tau\right)=\mathrm{f}\left(\alpha,\tau\right) \tag{1.8}$$

where 'u' is the unknown function, L and N are linear and nonlinear operators, and f is the source term. The correction functional for Eq.(1.8) is given as:

$$u_{n+1}\left(\alpha,\tau\right)=u_n\left(\alpha,\tau\right)+\int_0^x\lambda[Lu_n\left(\xi,\tau\right)+N\breve{u}_n\left(\xi,\tau\right)-f\left(\xi,\tau\right)]d\xi \tag{1.9}$$

here 'λ' is a general Lagrange multiplier that can be found by the variation theory and $\breve{u}_n$ is measured as a restricted variation $\delta u_n=0$.

Laplace Variational Iteration Method (LVIM)

'Laplace Variational Iteration Method' (LVIM) is the mixture of 'Laplace transform' and 'Variational Iteration Method' (VIM). The VIM has recently focused heavily on solving a broad variety of problems, including algebraic, differential, partial-differential, functional-delay, and integral–differential ones. The key component of this method is the construction of a correction functional employing a general Lagrange multiplier that has been carefully chosen so that its adjustment solution is superior to the initial assessment function. The purpose of this work is to broaden the applicability of LVIM to provide convergent answers for fractional differential equations. It is applied by several authors for solving many kinds of difficulties of applied mathematics. For instance, we can see the works of Abassy et al.,[22] Hammouch and Mekkaoui,[23] Arife and Yildirim[36] etc. Here, in this work, a new approach of Laplace Variational Iteration Method (LVIM) has been taken under consideration to find the solutions of problems of fractional heat like equations one, two and three dimensions.

We consider the following, a general fractional non-linear non-homogeneous partial differential equation with the initial situations of the system to illustrate the basic terminology of this method,

$$D_\tau^\vartheta u\left(\alpha,\tau\right)+\mathrm{L}u\left(\alpha,\tau\right)+\mathrm{N}u\left(\alpha,\tau\right)=\mathrm{f}\left(\alpha,\tau\right), \quad m-1<\vartheta\le m, m\in\mathbb{N}, \tag{1.10}$$

$$u^n\left(\alpha,0\right)=h_k\left(\alpha\right),\qquad n=0,1,2,3,\ldots,m-1, \tag{1.11}$$

where D_τ^ϑ is the ϑ order fractional Caputo derivative. L,N and f are same as acknowledged with (1.8).

Using Laplace transform[17] on (1.10), we get

$$\mathscr{L}[u(\alpha,\tau)]=\frac{1}{s^\vartheta}\sum_{n=0}^{m-1}s^{\vartheta-1-n}u^n(\alpha,0)+\frac{1}{s^\vartheta}\mathscr{L}[f(\alpha,\tau)] -\frac{1}{s^\vartheta}\mathscr{L}[Lu(\alpha,\tau)+Nu(\alpha,\tau)], \tag{1.12}$$

taking inverse Laplace transform, we have

$$u(\alpha,\tau)=\mathscr{L}^{-1}\left[\frac{1}{s^\vartheta}\sum_{n=0}^{m-1}s^{\vartheta-1-n}u^n(\alpha,0)+\frac{1}{s^\vartheta}\mathscr{L}[f(\alpha,\tau)]\right] -\mathscr{L}^{-1}\left[\frac{1}{s^\vartheta}\mathscr{L}[Lu(\alpha,\tau)+Nu(\alpha,\tau)]\right], \tag{1.13}$$

by differentiating (1.13), concerning τ, we get

$$\frac{\partial u(\alpha,\tau)}{\partial \tau} = \frac{\partial}{\partial t}\left\{\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\sum_{n=0}^{m-1} s^{\vartheta-1-n}u^{n}(\alpha,0)+\frac{1}{s^{\vartheta}}\mathscr{L}[f(\alpha,\tau)]\right]\right. \\ \left. -\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\mathscr{L}[Lu(\alpha,\tau)+Nu(\alpha,\tau)]\right]\right\}, \tag{1.14}$$

the above way has been adopted to be able to design the correction functional for (1.14) as

$$u_{n+1}(\alpha,\tau) = u_n(\alpha,\tau) \\ +\int_0^{\tau}\lambda\left[\frac{\partial u_n(\alpha,\varepsilon)}{\partial \varepsilon}-\frac{\partial}{\partial \varepsilon}\left\{\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\sum_{n=0}^{m-1} s^{\vartheta-1-n}u^{n}(\alpha,0)+\frac{1}{s^{\vartheta}}\mathscr{L}[f(\alpha,\tau)]\right]\right.\right. \\ \left.\left.-\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\mathscr{L}[Lu(\alpha,\varepsilon)+Nu(\alpha,\varepsilon)]\right]\right\}\right]d\varepsilon. \tag{1.15}$$

By variation theory, λ for (1.15) can be obtained as

$$1+\lambda|_{\varepsilon=\tau} = 0,$$

So,

$$\lambda = -1.$$

From (1.15), we get

$$u_{n+1}(\alpha,\tau)= u_n(\alpha,\tau) \\ -\int_0^{\tau}\left[\frac{\partial u_n(\alpha,\varepsilon)}{\partial \varepsilon}-\frac{\partial}{\partial \varepsilon}\left\{\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\sum_{n=0}^{m-1} s^{\vartheta-1-n}u^{n}(\alpha,0)+\frac{1}{s^{\vartheta}}\mathscr{L}[f(\alpha,\tau)]\right]\right.\right. \\ \left.\left.-\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\mathscr{L}[Lu(\alpha,\varepsilon)+Nu(\alpha,\varepsilon)]\right]\right\}\right]d\varepsilon \qquad ;n=0,1,2,\ldots$$

start with the preliminary iteration

$$u_0(\alpha,\tau) = u(\alpha,0)+\tau u_{\tau}(\alpha,0).$$

The exact solution will be given as

$$u(\alpha,\tau) = \lim_{n\to\infty} u_n(\alpha,\tau) \quad .$$

2. Main results

Applications of LVIM for Solving fractional heat-like equations

In this section, we consider three fractional heat like equations of one dimension (1-D), two dimensions (2-D) and three dimensions (3-D) respectively and find their solutions with the approach of LVIM. These exemplary problems are of high importance in Thermal engineering, Mechanical engineering, Chemical engineering, and various fields of Thermal Physics and Chemistry.

Further, the numerical and graphical discussion of the obtained results are also presented to highlight the behaviour of the solutions of the differential equations.

Problem 1
Let us examine the given one-dimensional fractional heat-like equation

$$D_t^{\vartheta}u(\alpha,\tau)= \frac{1}{2}\alpha^2\frac{\partial^2 u}{\partial \alpha^2} \qquad ;0<\vartheta\leq 1, \tag{2.1}$$

with the initial condition

$$u\left(\alpha,0\right)=\alpha^2. \tag{2.2}$$

Solution: Taking the Laplace transform of (2.1) and using the condition(2.2), we obtain

$$\mathscr{L}[u\left(\alpha,\tau\right)]=\frac{\alpha^2}{s}+\frac{1}{2s^{\vartheta}}x'^2\mathscr{L}\left[\frac{\partial^2 u}{\partial\alpha^2}\right], \tag{2.3}$$

applying inverse Laplace transform to (2.3), we get

$$u\left(\alpha,\tau\right)=\alpha^2+\mathscr{L}^{-1}\left[\frac{1}{2s^{\vartheta}}x^2\mathscr{L}\left[\frac{\partial^2 u}{\partial\alpha^2}\right]\right], \tag{2.4}$$

differentiating (2.4) concerning t, we have

$$\frac{\partial u}{\partial\tau}=\frac{\partial}{\partial\tau}\mathscr{L}^{-1}\left[\frac{1}{2s^{\vartheta}}\alpha^2\mathscr{L}\left[\frac{\partial^2 u}{\partial\alpha^2}\right]\right], \tag{2.5}$$

the correction functional for (2.5) with $\lambda=-1$ is given by

$$u_{n+1}\left(\alpha,\tau\right)=u_n\left(\alpha,\tau\right)-\int_0^t\left[\frac{\partial u_n(\alpha,\varepsilon)}{\partial\varepsilon}-\frac{\partial}{\partial\varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{2s^{\vartheta}}\alpha^2\mathscr{L}\left(\frac{\partial^2 u_n}{\partial\alpha^2}\right)\right\}\right]d\varepsilon.$$

The initial iteration is

$$u_0\left(\alpha,\tau\right)=u_0\left(\alpha,0\right)=\alpha^2, \tag{2.6}$$

then, we have

$$u_1\left(\alpha,\tau\right)=u_0\left(\alpha,\tau\right)-\int_0^\tau\left[\frac{\partial u_0(\alpha,\varepsilon)}{\partial\varepsilon}-\frac{\partial}{\partial\varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{2s^{\vartheta}}\alpha^2\mathscr{L}\left(\frac{\partial^2 u_0}{\partial\alpha^2}\right)\right\}\right]d\varepsilon, \tag{2.7}$$

$$u_1\left(\alpha,\tau\right)=\alpha^2+\alpha^2\frac{t^{\vartheta}}{\Gamma(\vartheta+1)}, \tag{2.8}$$

$$u_2\left(\alpha,\tau\right)=u_1\left(\alpha,\tau\right)-\quad\int_0^\tau\left[\frac{\partial u_1(\alpha,\varepsilon)}{\partial\varepsilon}-\frac{\partial}{\partial\varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{2s^{\vartheta}}\alpha^2\mathscr{L}\left(\frac{\partial^2 u_1}{\partial\alpha^2}\right)\right\}\right]d\varepsilon \tag{2.9}$$

$$u_2\left(\alpha,\tau\right)=\alpha^2+\alpha^2\frac{\tau^{\vartheta}}{\Gamma(\vartheta+1)}+\alpha^2\frac{\tau^{2\vartheta}}{\Gamma(2\vartheta+1)}, \tag{2.10}$$

then the term given below in the successive approximation is

$$u_n\left(\alpha,\tau\right)=\alpha^2\left[1+\frac{\tau^{\vartheta}}{\Gamma(\vartheta+1)}+\frac{\tau^{2\vartheta}}{\Gamma(2\vartheta+1)}+\frac{\tau^{3\vartheta}}{\Gamma(3\vartheta+1)}\cdots\right]. \tag{2.11}$$

Therefore, the solution is given by

$$u\left(\alpha,\tau\right)=\lim_{n\to\infty}u_n\left(\alpha,\tau\right)\quad=\alpha^2E_{\vartheta}\left(\tau^{\vartheta}\right), \tag{2.12}$$

where $E_{\vartheta}(.)$, is the well-known Mittag-Leffler function,[27] defined in (1.7).

Special Case

Letting $\vartheta=1$, then

$$u(\alpha,\tau)=\alpha^2E_1\left(\tau^1\right)=\alpha^2e^{\tau}.$$

Numerical and Graphical discussion for Problem 1.

In this part we found a record for numerical explanation of Eq.(2.11) and plot some graphs for different values of $\vartheta = 0.25, 0.5, 0.75, 1$ (see Table 1, Table 2, Table 3, Table 4).

Figs. 1, 2, 3, and 4 show the estimated solution ofEq.(2.1) for different fix standards of ϑ and shows the exponential behaviour of the solution.

Table 1. For fix $\vartheta = 0.25$.

τ	$\alpha = \mathbf{1}$	$\alpha = \mathbf{3}$	$\alpha = \mathbf{5}$	$\alpha = \mathbf{7}$	$\alpha = \mathbf{9}$
0	1	9	25	49	81
2	5.737677	51.639095	143.441931	281.146185	464.751857
4	7.894520	71.050685	197.363015	386.831509	639.456169
6	9.661923	86.957309	241.548082	473.434241	782.615787
8	11.222737	101.004635	280.568430	549.914123	909.041714
10	12.648798	113.839182	316.219951	619.791104	1024.552642

Table 2. For fix $\vartheta = 0.5$.

τ	$\alpha = \mathbf{1}$	$\alpha = \mathbf{3}$	$\alpha = \mathbf{5}$	$\alpha = \mathbf{7}$	$\alpha = \mathbf{9}$
0	1	9	25	49	81
2	26.893845	60.511151	168.086532	329.449602	544.600364
4	53.099122	119.473025	331.869514	650.464247	1075.257225
6	83.279063	187.377893	520.494149	1020.16853	1686.401044
8	116.852302	262.917679	730.326888	1431.440702	2366.259119
10	153.426279	345.209128	958.914244	1879.471919	3106.882153

Table 3. For fix $\vartheta = 0.75$.

τ	$\alpha = \mathbf{1}$	$\alpha = \mathbf{3}$	$\alpha = \mathbf{5}$	$\alpha = \mathbf{7}$	$\alpha = \mathbf{9}$
0	1	9	25	49	81
2	27.294236	61.412032	170.588978	334.354398	552.708290
4	75.8864748	170.744568	474.290468	929.609317	1536.701116
6	153.315341	344.959518	958.220884	1878.112934	3104.635665
8	261.677129	588.773541	1635.482059	3205.544836	5298.961871
10	402.654942	905.973621	2516.593392	4932.523048	8153.762590

Table 4. For fix $\vartheta = 1$.

τ	$\alpha = \mathbf{1}$	$\alpha = \mathbf{3}$	$\alpha = \mathbf{5}$	$\alpha = \mathbf{7}$	$\alpha = \mathbf{9}$
0	1	9	25	49	81
2	25.33	57	158.33	310.33	513
4	94.66	213	591.66	1159.66	1917
6	244	549	1525	2989	4941
8	504.33	1137	3158.33	6190.33	10233
10	910.66	2049	5691.66	11 155.66	18441

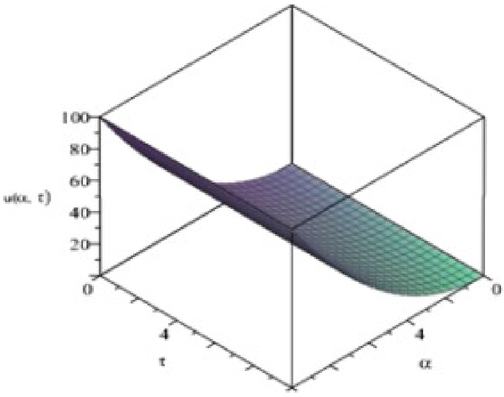

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Fig. 1. $\vartheta = 0.25$.

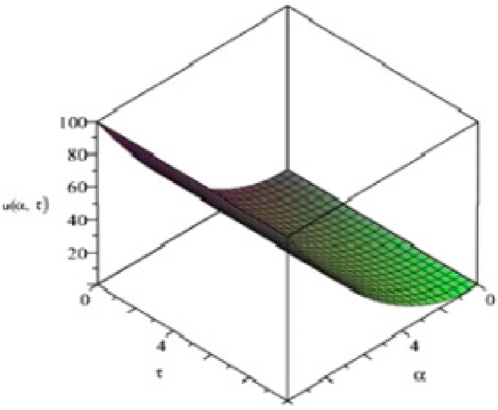

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Fig. 2. $\vartheta = 0.5$.

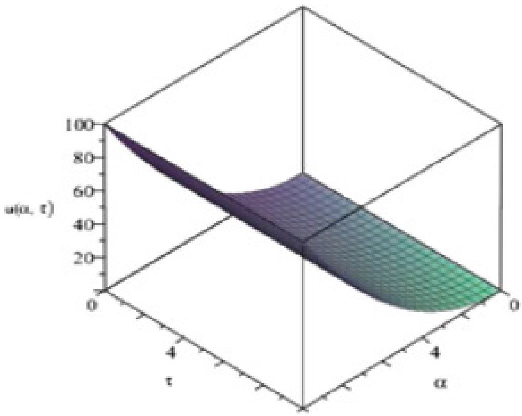

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Fig. 3. $\vartheta = 0.75$.

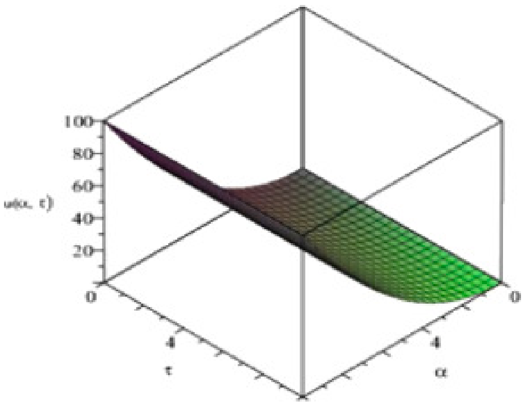

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Fig. 4. $\vartheta = 1$.

Problem 2

Let us examine the given two-dimensional fractional heat-like equation

$$D_\tau^\vartheta u(\alpha, \beta, \tau) = \frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2}, \qquad 0 < \vartheta \leq 1, \tag{2.13}$$

with initial condition

$$u(\alpha, \beta, 0) = sin\alpha \quad sin\beta \quad . \tag{2.14}$$

Solution: Taking the Laplace transform of (2.13) and using the condition specified by (2.14), we find,

$$\mathscr{L}[u(\alpha, \beta, \tau)] = \frac{sin\alpha \quad sin\beta}{s} + \frac{1}{s^\vartheta} \mathscr{L}\left[\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2}\right], \tag{2.15}$$

applying inverse Laplace transform, we have

$$u(\alpha, \beta, \tau) = sin\alpha \quad sin\beta \quad + \mathscr{L}^{-1}\left[\frac{1}{s^\vartheta} \mathscr{L}\left[\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2}\right]\right], \tag{2.16}$$

differentiating (2.16) concerning τ, we have

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial \tau}\mathscr{L}^{-1}\left[\frac{1}{s^{\vartheta}}\mathscr{L}\left[\frac{\partial^2 u}{\partial \alpha^2} + \frac{\partial^2 u}{\partial \beta^2}\right]\right]. \tag{2.17}$$

The correction functional for (2.17) with $\lambda = -1$ is given by

$$u_{n+1}(\alpha,\beta,\tau) = u_n(x',y',\tau) - \int_0^{\tau}\left[\frac{\partial u_n(\alpha,\beta,\varepsilon)}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{s^{\vartheta}}\mathscr{L}\left(\frac{\partial^2 u_n}{\partial \alpha^2} + \frac{\partial^2 u_n}{\partial \beta^2}\right)\right\}\right]d\varepsilon. \tag{2.18}$$

The initial iteration

$$u_0(\alpha,\beta,\tau) = u_0(\alpha,\beta,0) = sin\alpha \quad sin\beta \quad , \tag{2.19}$$

then, we have

$$u_1(\alpha,\beta,\tau) = u_0(\alpha,\beta,\tau) - \int_0^{\tau}\left[\frac{\partial u_0(\alpha,\beta,\varepsilon)}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{s^{\vartheta}}\mathscr{L}\left(\frac{\partial^2 u_0}{\partial \alpha^2} + \frac{\partial^2 u_0}{\partial \beta^2}\right)d\varepsilon\right\}\right]d\varepsilon \tag{2.20}$$

$$u_1(\alpha,\beta,\tau) = sin\alpha \quad sin\beta \quad - 2sin\alpha \quad sin\beta \quad \frac{\tau^{\vartheta}}{\Gamma(\vartheta+1)}, \tag{2.21}$$

$$u_2(\alpha,\beta,\tau) = u_1(\alpha,\beta,\tau) - \int_0^{\tau}\left[\frac{\partial u_1(\alpha,\beta,\varepsilon)}{\partial \varepsilon} - \frac{\partial}{\partial \varepsilon}\mathscr{L}^{-1}\left\{\frac{1}{s^{\vartheta}}\mathscr{L}\left(\frac{\partial^2 u_1}{\partial \alpha^2} + \frac{\partial^2 u_1}{\partial y'^2}\right)\right\}\right]d\varepsilon \tag{2.22}$$

$$u_2(\alpha,\beta,\tau) = sin\alpha \quad sin\beta \quad - 2sin\alpha \quad sin\beta \quad \frac{\tau^{\vartheta}}{\Gamma(\vartheta+1)} + 4sin\alpha \quad sin\beta \quad \frac{\tau^{2\vartheta}}{\Gamma(2\vartheta+1)}, \tag{2.23}$$

then the term given below in the successive approximation is

$$u_n(\alpha,\beta,\tau) = sin\alpha \quad sin\beta \quad \left[1 + \frac{(-2\tau^{\vartheta})}{\Gamma(\vartheta+1)} + \frac{(-2\tau^{\vartheta})^2}{\Gamma(2\vartheta+1)} + \frac{(-2\tau^{\vartheta})^3}{\Gamma(3\vartheta+1)}\ldots\right]. \tag{2.24}$$

Therefore, the solution is given by

$$u(\alpha,\beta,\tau) = \lim_{n\to\infty} u_n(\alpha,\beta,\tau) \quad = sin\alpha \quad sin\beta \quad E_{\vartheta}\left(-2\tau^{\vartheta}\right), \tag{2.25}$$

where $E_{\vartheta}(.)$, is the well-known Mittag-Leffler function.

Special Case

letting $\vartheta = 1$, then

$$u(\alpha,\beta,\tau) = sin\alpha \quad sin\beta \quad E_1\left(-2\tau^1\right) = e^{-2\tau} sin\alpha \quad sin\beta \quad .$$

Numerical and Graphical discussion for Problem2.

In this part we found a record for numerical explanation of Eq.(2.24) and plot some graphs for different values of $\vartheta = 0.25, 0.5, 0.75, 1$ (see Table 5, Table 6, Table 7, Table 8).

Figs. 5, 6, 7, and 8 show the estimated solution of (2.24) for different standards of ϑ and shows the sine and cosine wave behaviour of the solution.

Table 5. For fix $\vartheta = 0.25$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	0.174524	0.052335	0.871557	0.121869	0.156434
2	−0.172432	−0.517153	−0.861221	−1.204240	−1.545793
4	−0.309144	−0.927056	−1.543839	−2.158741	−2.771012
6	−0.432256	−1.296243	−2.158651	−3.018429	−3.874529
8	−0.547144	−1.640767	−2.732392	−3.820687	−4.904327
10	−0.656209	−1.967829	−3.277051	−4.582281	−5.881928

Table 6. For fix $\vartheta = 0.5$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	0.174524	0.052335	0.871557	0.121869	0.156434
2	−0.195695	0.586847	−0.977283	−1.366530	−1.754111
4	−0.622312	−1.866179	−3.107772	−4.345579	−5.578092
6	−1.203769	−3.609841	−6.011515	−8.405865	−10.789974
8	−1.912004	−5.733685	−9.548379	−13.351441	−17.138236
10	−2.730308	−8.187597	−13.634911	−19.06561	−24.47308

Table 7. For fix $\vartheta = 0.75$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	0.174524	0.052335	0.871557	0.121869	0.156434
2	−0.158411	−0.475041	−0.791092	−1.106179	−1.419918
4	−0.909123	−2.726263	−4.540082	−6.348369	−8.148922
6	−2.442173	−7.323544	−12.195993	−17.053583	−21.89039
8	−4.869943	−14.603897	−24.320058	−34.006588	−43.651687
10	−8.274852	−24.814475	−41.323865	−57.782909	−74.171553

Table 8. For fix $\vartheta = 1$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	0.174524	0.052335	0.871557	0.121869	0.156434
2	−0.098896	−0.296570	−0.493882	−0.690592	−0.886461
4	−1.052961	−3.157602	−5.258396	−7.352783	−9.438212
6	−3.909339	−11.723254	−19.522886	−27.298732	−35.041320
8	−9.942054	−29.814049	−49.649721	−69.424902	−89.115500
10	−20.110989	−60.308466	−100.432467	−140.434106	−180.264648

Table 9. For fix $\vartheta = 0.25$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	1	81	625	2401	6561
2	5.737677	464.751857	3586.048285	13776.16309	37644.90047
4	7.8945206	639.456169	4934.075381	18954.74399	51795.94972
6	9.661923	782.615787	6038.702061	23198.27783	63391.87874
8	11.2227372	909.0417147	7014.210761	26945.79205	73632.37889
10	12.648798	1024.552642	7905.498778	30369.76411	82988.76398

Table 10. For fix $\vartheta = 0.5$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	1	81	625	2401	6561
2	6.723461	544.600364	4202.163303	16143.03054	44112.62948
4	13.274780	1075.257225	8296.737850	31872.74812	87095.83525
6	20.819765	1686.401044	13012.35374	49988.25812	1.365984846 10^5
8	29.213075	2366.259119	18258.17221	70140.59438	1.916669886 10^5
10	38.356569	3106.882153	23972.85612	92094.12405	2.516574544 10^5

Table 11. For fix $\vartheta = 0.75$.

	1	**3**	**5**	**7**	**9**
0	1	81	625	2401	6561
2	6.885719	557.743276	4303.57466	16532.61242	45177.20538
4	19.0761595	1545.168919	11922.59969	45801.85896	1.251586825 10^5

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
6	38.4705302	3116.112951	24044.08142	92367.74317	2.504051490 10^5
8	65.5950983	5313.202962	40996.93643	1.574938311 10^5	4.303694400 10^5
10	100.871581	8170.598080	63044.73826	2.421926665 10^5	6.618184444 10^5

Table 12. For fix $\vartheta = 1$.

τ	$\alpha = 1$	$\alpha = 3$	$\alpha = 5$	$\alpha = 7$	$\alpha = 9$
0	1	81	625	2401	6561
2	6.3333	513	3958.3333	15206.3333	41553
4	23.6666	1917	14791.6666	56,823.6666	155277
6	61	4941	38125	146461	400221
8	126.3333	10233	78958.3333	303326.3333	828873
10	227.6666	18441	142291.66	546627.66	1493721

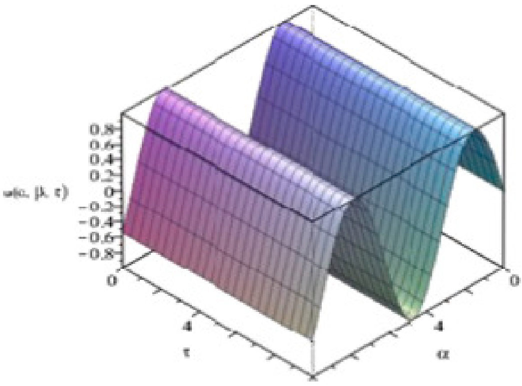

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Fig. 5. $\vartheta = 0.25$.

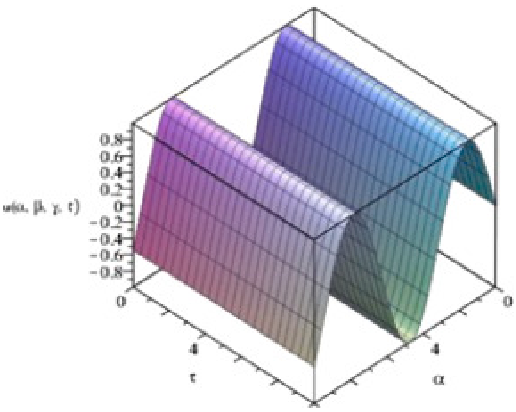

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Fig. 6. $\vartheta = 0.5$.

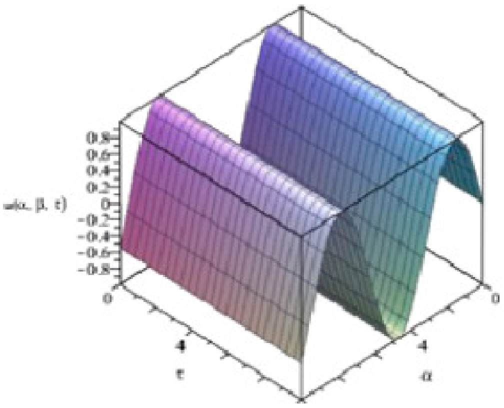

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Fig. 7. $\vartheta = 0.75$.

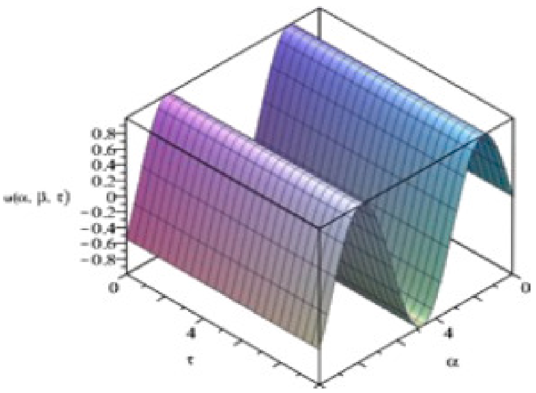

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Fig. 8. $\vartheta = 1$.

Problem 3

Let us examine the given three-dimensional fractional heat-like equation

$$D_\tau^\vartheta u(\alpha,\beta,\gamma,\tau)= \alpha^4\beta^4\gamma^4 + \frac{1}{36}\left[\alpha^2\frac{\partial^2 u}{\partial\alpha^2} + \beta^2\frac{\partial^2 u}{\partial\beta^2} + \gamma^2\frac{\partial^2 u}{\partial\gamma^2}\right], \qquad 0 < \vartheta \le 1, \tag{2.26}$$

with the initial condition

$$u(\alpha,\beta,\gamma,0) = 0. \tag{2.27}$$

Solution: Taking the Laplace transform of (2.26) and using the condition specified by (2.27), we find,

$$\mathscr{L}[u(\alpha,\beta,\gamma,\tau)]= \frac{1}{s^\vartheta}\mathscr{L}\left(\alpha^4\beta^4\gamma^4\right)+\frac{1}{36s^\vartheta}\mathscr{L}\left[\alpha^2\frac{\partial^2 u}{\partial\alpha^2} + \beta^2\frac{\partial^2 u}{\partial\beta^2} + \gamma^2\frac{\partial^2 u}{\partial\gamma^2}\right], \tag{2.28}$$

applying inverse Laplace transform, we get

$$u(\alpha,\beta,\gamma,\tau) = \left[\left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^\vartheta}{\Gamma(\vartheta+1)}\right] + \mathscr{L}^{-1}\left[\frac{1}{36s^\vartheta}\mathscr{L}\left[\alpha^2\frac{\partial^2 u}{\partial\alpha^2} + \beta^2\frac{\partial^2 u}{\partial\beta^2} + \gamma^2\frac{\partial^2 u}{\partial\gamma^2}\right]\right], \tag{2.29}$$

differentiating (2.29) concerning τ, we have

$$\frac{\partial u}{\partial\tau} = \left(\alpha^4\beta^4\gamma^4\right)\frac{\vartheta\tau^{\vartheta-1}}{\Gamma(\vartheta+1)} + \frac{\partial}{\partial t}\left\{\mathscr{L}^{-1}\left[\frac{1}{36s^\vartheta}L\left[\alpha^2\frac{\partial^2 u}{\partial\alpha^2} + \beta^2\frac{\partial^2 u}{\partial\beta^2} + \gamma^2\frac{\partial^2 u}{\partial\gamma^2}\right]\right]\right\}. \tag{2.30}$$

The correction functional for (2.30) with $\lambda = -1$ is given by

$$u_{n+1}(\alpha,\beta,\gamma,\tau) = u_n(\alpha,\beta,\gamma,\tau) - \int_0^\tau\left[\frac{\partial u_n(\alpha,\beta,\gamma,\varepsilon)}{\partial\varepsilon} - \left(\alpha^2\beta^2\gamma^2\right)\frac{\vartheta\tau^{\vartheta-1}}{\Gamma(\vartheta+1)} - \frac{\partial}{\partial\varepsilon}\left\{\mathscr{L}^{-1}\left[\frac{1}{36s^\vartheta}L\left[\alpha^2\frac{\partial^2 u_n}{\partial\alpha^2} + \beta^2\frac{\partial^2 u_n}{\partial\beta^2} + \gamma^2\frac{\partial^2 u_n}{\partial\gamma^2}\right]\right]\right\}\right]d\varepsilon \tag{2.31}$$

and the initial iteration

$$u_0(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^\vartheta}{\Gamma(\vartheta+1)}, \tag{2.32}$$

then, we have

$$u_1(\alpha,\beta,\gamma,\tau) = u_0(\alpha,\beta,\gamma,\tau) - \int_0^\tau\left[\frac{\partial u_0(\alpha,\beta,\gamma,\varepsilon)}{\partial\varepsilon} - \left(\alpha^2\beta^2\gamma^2\right)\frac{\vartheta\tau^{\vartheta-1}}{\Gamma(\alpha+1)} - \frac{\partial}{\partial\varepsilon}\left\{\mathscr{L}^{-1}\left[\frac{1}{36s^\vartheta}\mathscr{L}\left[\alpha^2\frac{\partial^2 u_0}{\partial\alpha^2} + \beta^2\frac{\partial^2 u_0}{\partial\beta^2} + \gamma^2\frac{\partial^2 u_0}{\partial\gamma^2}\right]\right]\right\}\right]d\varepsilon \tag{2.33}$$

$$u_1(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^\vartheta}{\Gamma(\vartheta+1)} + \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^{2\vartheta}}{\Gamma(2\vartheta+1)}, \tag{2.34}$$

$$u_2(\alpha,\beta,\gamma,\tau) = u_1(\alpha,\beta,\gamma,\tau) - \int_0^\tau\left[\frac{\partial u_1(\alpha,\beta,\gamma,\varepsilon)}{\partial\varepsilon} - \left(\alpha^2\beta^2\gamma^2\right)\frac{\alpha\tau^{\alpha-1}}{\Gamma(\alpha+1)} - \frac{\partial}{\partial\varepsilon}\left\{\mathscr{L}^{-1}\left[\frac{1}{36s^\vartheta}\mathscr{L}\left[\alpha^2\frac{\partial^2 u_1}{\partial\alpha^2} + \beta^2\frac{\partial^2 u_1}{\partial\beta^2} + \gamma^2\frac{\partial^2 u_1}{\partial\gamma^2}\right]\right]\right\}\right]d\varepsilon \tag{2.35}$$

$$u_2(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^\vartheta}{\Gamma(\vartheta+1)} + \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^{2\vartheta}}{\Gamma(2\vartheta+1)} + \left(\alpha^4\beta^4\gamma^4\right)\frac{\tau^{3\vartheta}}{\Gamma(3\vartheta+1)}, \tag{2.36}$$

then the term given below in the successive approximation is

$$u_n(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\left[\frac{(\tau^\vartheta)}{\Gamma(\vartheta+1)} + \frac{(\tau^\vartheta)^2}{\Gamma(2\vartheta+1)} + \frac{(\tau^\vartheta)^3}{\Gamma(3\vartheta+1)}\cdots\right]. \tag{2.37}$$

Therefore, the solution is given by

$$u(\alpha,\beta,\gamma,\tau) = \lim_{n\to\infty} u_n(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\left[E_\vartheta\left(\tau^\vartheta\right) - 1\right], \tag{2.38}$$

here $E_\vartheta(.)$, is the well-known Mittag-Leffler function.

Special Case

Letting $\vartheta = 1$, then

$$u(\alpha,\beta,\gamma,\tau) = \left(\alpha^4\beta^4\gamma^4\right)\left[E_1\left(\tau^1\right) - 1\right] = \left(\alpha^4\beta^4\gamma^4\right)\left(e^\tau - 1\right). \tag{2.39}$$

Numerical and Graphical discussion for Problem3.

In this part we found a record for numerical explanation of Eq.(2.37) and plot some graphs for different values of $\vartheta = 0.25, 0.5, 0.75, 1$ (see Table 9, Table 10, Table 11, Table 12).

Figs. 9, 10, 11, and 12 show the estimated solution ofEq.(2.37) for different standards of ϑ and shows the specific exponential behaviour of the solution.

The results obtained in (2.12), (2.25), (2.38) are comparable with the results obtained by Natural transform decomposition method[37] and Optimal Homotopy Analysis method[38] and are more refined in view of the linearization, discretization, and perturbation, which are not needed in LVIM. Although, due to lack of space, we are not presenting the comparison in this manuscript but for the purpose one may refer.[37,38]

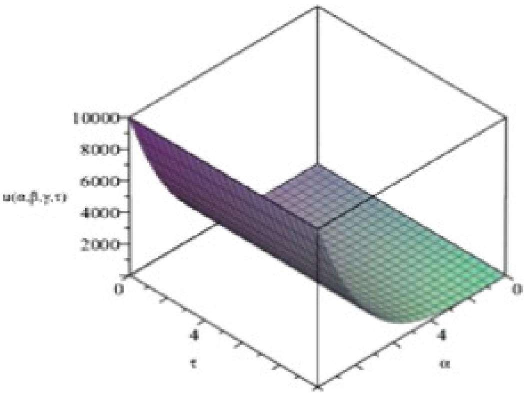

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Fig. 9. $\vartheta = 0.25$.

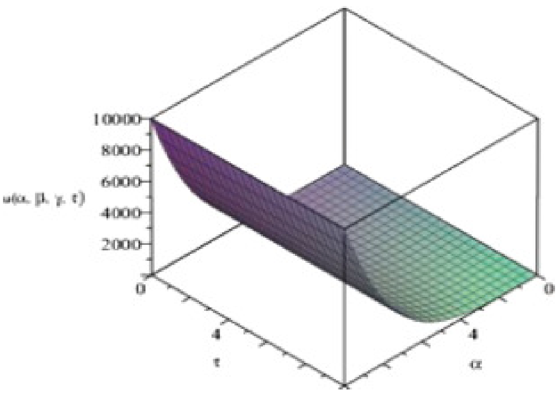

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Fig. 10. $\vartheta = 0.5$.

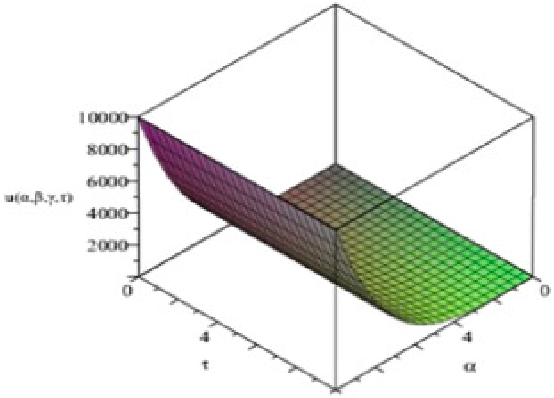

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Fig. 11. $\vartheta = 0.75$.

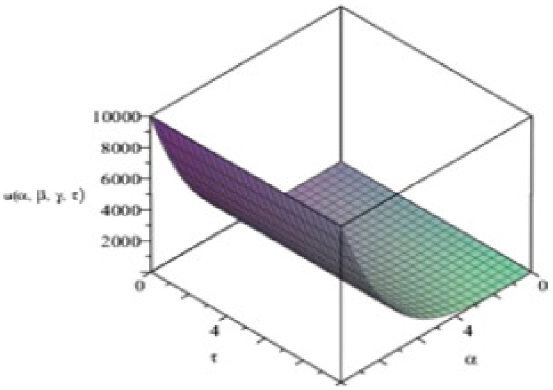

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Fig. 12. $\vartheta = 1$.

3. Conclusion

In the presented manuscript, the 'Laplace variational iteration method' is effectively implemented for the 1-D, 2-D, and 3-D fractional heat like differential equations, wherever we apply the fractional Caputo derivative. The analytical results in particularly have generated in terminologies of a power series that converges to the exact solutions. The graphical consequences of the analysis are also acknowledged. Fig. 1, Fig. 2, Fig. 3, Fig. 4, Fig. 5, Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, Fig. 11, Fig. 12 show the behaviour of the estimated solution of the Problems 1, 2, and 3 respectively with suitable parametric values. The exemplary problems considered here are very popular and important in the domains of Mechanical, Thermal and Chemical engineering, Thermal Physics and chemical reactions. Hence, it is assumed that the results obtained here may be very useful for research and industrial use.

CRediT authorship contribution statement

Alok Bhargava: Creation of the work, Design of the work, Handled the analysis. **Deepika Jain:** Design of the work, Handled the analysis, Conceptualized, Doublechecked the Analysis part. **D.L. Suthar:**

Manuscript's drafting or critical revision for important intellectual content.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

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