Swami Keshvanand Institute of Technology,
Management & Gramothan

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Affiliated to Rajasthan Technical University, Kota

1.1.2 Midterm Paper and Solution (Sample)

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Swami Keshvanand Institute of Technology,
Management & Gramothan, Jaipur
II Mid Term Examination, 2020-21

A

B.Tech./ Semester - VI
Course: Mechanical Vibrations
Time: 1½ Hours

Branch: Mechanical Engg.
Course Code: 6ME4-03
Maximum Marks: 50

Attempt all questions. Each question carries 10 marks.

1. A vibrating system having mass 2 kg is suspended by a spring of stiffness 1250 N/m and it is put to harmonic excitation of 20 N. Assuming viscous damping with c = 42 N-s/m, determine:

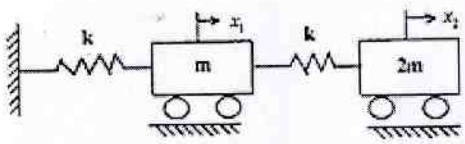
(i) The resonant frequency

(ii) The amplitude at resonance

(iii) The frequency corresponding to the peak amplitude

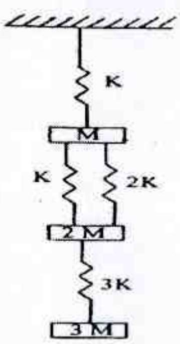
(iv) Peak amplitude **(4 x 2 ½ marks)**

2. Determine both the natural frequencies and mode shapes for the spring mass system shown in figure below. Take **m** = 4 kg, and **k** = 800 N/m.



(10 marks)

3. Use Stodola's method to find the fundamental natural frequency of the system shown in figure below.



(10 marks)

4. Write the wave equation (no need to derive) for torsional vibrations of a shaft treated as a continuous system. Solve the wave equation to get the frequency equation and draw the first three mode shapes for fixed-fixed boundary conditions. **(10 marks)**

5. **(a)** Write short notes on noise control strategies. **(5 marks)**

(b) In an industry the SPL of ambient sound near a worker is 88 dB when all machines are turned off near him. There are 4 machines near that worker producing 90 dB, 86 dB, 92 dB and 89 dB respectively. Find the total SPL at the position of worker due to both the ambient sound and the machines. **(5 marks)**

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(B)

Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur
II Mid Term Examination, 2020-21

B.Tech./ Semester - VI | **Branch: Mechanical Engg.**
Course: Mechanical Vibrations | **CourseCode: 6ME4-03**
Time: 1½ Hours | **Maximum Marks: 50**

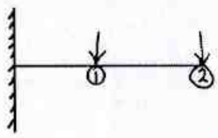
Attempt all questions. Each question carries 10 marks.

1. A radio set of 100 N weight must be isolated from a machine vibrating with amplitude of 0.06 mm at 600 cpm. The set is mounted on 4 parallel isolators, each having a spring scale of 3.2 kN/m and damping coefficient of 0.4 kN-s/m. Determine: **(a)** Amplitude of vibration of radio, **(b)** Dynamic load on each isolator due to vibration. **(6 + 4 marks)**

2. (i) An engine weighing 400 N was found to vibrate violently when it was run at 3000 rpm. Design a vibration absorber for the engine, so that the nearest resonant frequency of the combined system is at least 25% away from the operating speed. **(5 marks)**

(ii) A rotor has a mass of 9 kg mounted midway on a 22 mm diameter horizontal shaft supported at the ends by two short bearings which are 1m apart. The shaft rotates at 2500 rpm. If the center of mass of the rotor is 0.13 mm away from the geometric center of the rotor due to certain manufacturing defects, find the amplitude of steady state vibrations and the critical speed. Take E = 210 GN/m^2. **(5 marks)**

3. Figure below shows a cantilever beam with point loads at 2 locations. The loads at locations 1 and 2 are 1000 N and 800 N respectively. The distances of locations 1 and 2 from the fixed end are 1 m and 2 m respectively. Take $E = 2 \times 10^{11}$ N/m^2 and cross section moment of inertia of the beam $I = 4 \times 10^{-7}$ m^4. Using Rayleigh's method, find the fundamental natural frequency of transverse vibrations.



(10 marks)

4. With the help of a neat diagram, derive the wave equation of motion for longitudinal vibrations of an elastic bar by assuming it as a continuous system. Then solve the wave equation to get the general solution. ***Note:*** No need to draw the mode shapes. **(10 marks)**

5. Write short notes on the following:

(i) Frequency dependent human response to sound **(3 marks)**
(ii) Industrial noise control strategies **(7 marks)**

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Swami Keshvanand Institute of Technology,
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II Mid Term Examination, 2020-21

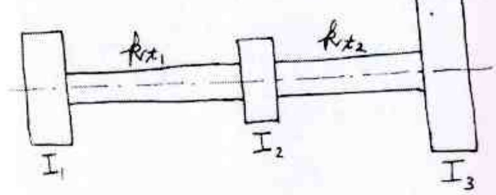
B.Tech./ Semester - VI | **Branch: Mechanical Engg.**
Course: Mechanical Vibrations | **Course Code: 6ME4-03**
Time: 1½ Hours | **Maximum Marks: 50**

Attempt all questions. Each question carries 10 marks.

1. A machine operates at a constant speed of 200 r.p.m. and is mounted on four parallel springs having stiffness of 50 kN/m each. Damping ratio is 0.2. Rotating parts are well balanced. The reciprocating part weighs 20 kg. The stroke length is 0.2 m. Find the dynamic amplitude of vertical motion and the phase difference between the motion and excitation force. The weight of the machine is 600 kg. **(10 marks)**

2. A vertical steel shaft of 10 mm diameter is held in long bearings 1 m apart and carries at its middle a disc of mass 10 kg. The eccentricity of the center of gravity of the disc from the center of the rotor is 0.25 mm. The modulus of Elasticity of the shaft material is 200 GN/m^2 and the permissible stress is 75 MN/m^2. Find: (i) The critical speed of the shaft and (ii) The range of speed over which it is unsafe to run the shaft. Neglect the mass of the shaft. **(3 + 7 marks)**

3. Determine the natural frequencies and mode shapes for a three degree of freedom torsional system shown in figure below. The system consists of 3 rotors of mass moments of inertia I_1 = 3 kg-m^2, I_2 = 2 kg-m^2, and I_3 = 4 kg-m^2. They are connected by two shafts of stiffnesses k_{t1} = 6 N-m/rad, and k_{t2} = 9 N-m/rad.



(10 marks)

4. Derive the wave equation for transverse vibrations of a tight string. Solve the wave equation to get the general solution. ***Note:*** No need to draw the mode shapes. **(10 marks)**

5. **(i)** Define sound pressure level and sound intensity level and derive the relationship between the two. **(1+1+3 marks)**

   (ii) A complex machine comprising of four rotary components acts as a noise source while it is put to work at a factory. There is some background noise at the factory too. The SPL of various components of the machine measured while it was working at this factory are: component 1 creates 90 dB SPL, component 2 creates 95 dB SPL, component 3 creates 85 dB SPL, and component 4 creates 80 dB SPL. When the machine is turned off, the SPL at the same place is 75 dB. Determine the total SPL of the overall machine independent of the background noise. **(5 marks)**

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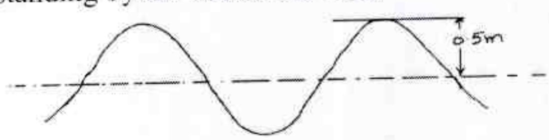
Swami Keshvanand Institute of Technology, Management & Gramothan, Jaipur
II Mid Term Examination, 2020-21

P

B.Tech./ Semester - VI | **Branch: Mechanical Engg.**
Course: Mechanical Vibrations | **CourseCode:6ME4-03**
Time: 1½ Hours | **Maximum Marks: 50**

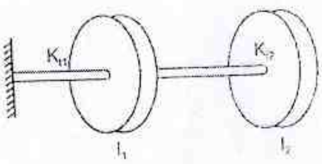
Attempt all questions. Each question carries 10 marks.

1. A fully loaded automobile is moving at a speed of 100 km/hr on a road with sinusoidal profile with a wavelength of 5 m. The vertical measurement of each crest of the sinusoidally shaped road above the horizontal mean line is 0.5 m (see figure shown below). The suspension springs of the automobile have a total stiffness of 250 kN/m. At full capacity (950 kg) of the automobile, it was found that the damping ratio is 0.45. Find the absolute amplitude of the vehicle as observed by an external observer standing by the side of the road.



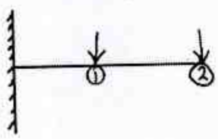
(10 marks)

2. Determine the natural frequencies and normal modes (i.e. mode shapes) of the torsional system shown in figure. Given: K_{t1} = 15 N-m/rad, K_{t2} = 25 N-m/rad, I_1 = 15 kg-m^2, I_2 = 20 kg-m^2.



(10 marks)

3. Figure below shows a cantilever beam with point loads at 2 locations. The loads at locations 1 and 2 are 800 N and 500 N respectively. The distances of locations 1 and 2 from the fixed end are 0.5 m and 1 m respectively.
Take E = 2 x 10^{11} N/m^2 and cross section moment of inertia of the beam I = 4 x 10^{-7} m^4. Using Stodola's method, find the fundamental natural frequency of transverse vibrations.



(10 marks)

4. Write the wave equation (no need to derive) for transverse vibrations of a tightly stretched string treated as a continuous system. Solve the wave equation to get the frequency equation and draw the first three mode shapes for fixed-fixed boundary conditions. **(10 marks)**

5. Write short notes on the following:
   (i) Auditory effects of noise **(3 marks)**
   (ii) Non-auditory effects of noise **(3 marks)**
   (iii) Major sources of noise **(4 marks)**

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1.)

Given: $m = 2\,kg.$; $k = 1250\ N/m$

$F_0 = 20\,N$; $c = 42\ N\text{-}s/m.$

$2\tfrac{1}{2} \times 4$

(i) $\omega_n = \sqrt{\dfrac{k}{m}} = 25\ rad/s$ (Ans.)

(ii) $$Y_0 = \frac{Y_{st}}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$$ $\Rightarrow$ at $r=1$, and taking $Y_{st} = \dfrac{F_0}{k}$

$$\Rightarrow Y_0 = \frac{F_0}{k(2\xi)} = \frac{F_0}{2k\left(\frac{c}{c_c}\right)}$$

$$= \frac{F_0}{2k\,\frac{c}{2\sqrt{km}}} = \frac{F_0}{\sqrt{\frac{k}{m}}\cdot c} = \frac{F_0}{c\cdot\omega_n}$$

$= 0.019$ ~~m~~ or $19\,mm.$ (Ans.)

(iii) $\omega_p = \omega_n\sqrt{1-2\xi^2} = 25\sqrt{1-2\xi^2}$

$$\xi = \frac{c}{c_c} = \frac{42}{2m\omega_n} = 0.42$$

$\Rightarrow \omega_p = 20.11\ rad/s.$ (Ans.)

(iv) $$\frac{Y_p}{Y_{st}} = \frac{1}{\sqrt{(1-r_p^2)^2 + (2\xi r_p)^2}}$$; where $r_p = \dfrac{\omega_p}{\omega_n} = \sqrt{1-2\xi^2}$

$$\Rightarrow \frac{Y_p}{\left(\frac{F_0}{k}\right)} = \frac{1}{2\xi\sqrt{1-\xi^2}} \Rightarrow Y_p = \frac{0.016}{2\times 0.42\sqrt{1-0.42^2}}$$

$= 0.021\,m$ or $21\,mm.$ (Ans.)

2.)

$$\left.\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 + k_2(x_2-x_1) \\ m_2\ddot{x}_2 &= -k_2(x_2-x_1) \end{aligned}\right\} \Rightarrow \text{Rearranging in matrix form:-}$$

$$\Rightarrow \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} (k_1+k_2) & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow (1) \qquad 2$$

Assuming sol. as:- $\left.\begin{aligned} x_1 &= X_1 \cdot \sin\omega t \\ x_2 &= X_2 \cdot \sin\omega t \end{aligned}\right\} \Rightarrow$ we get $\Rightarrow \begin{aligned} \ddot{x}_1 &= -\omega^2 X_1 \cdot \sin\omega t \\ \ddot{x}_2 &= -\omega^2 X_2 \cdot \sin\omega t \end{aligned}$

$\Rightarrow$ using these in matrix eq. & simplifying, we get:-

$$\begin{bmatrix} (k_1+k_2-m_1\omega^2) & -k_2 \\ -k_2 & k_2-m_2\omega^2 \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \longrightarrow (2)$$

$\Rightarrow$ For Non-Trivial sol. :-

$$\begin{vmatrix} (k_1+k_2-m_1\omega^2) & -k_2 \\ -k_2 & (k_2-m_2\omega^2) \end{vmatrix} = 0 \qquad 2$$

Using $m_1 = 4\,kg.$, $m_2 = 8\,kg.$, $k_1 = k_2 = 800$ N/m

$$\Rightarrow \begin{vmatrix} 1600-4\omega^2 & -800 \\ -800 & 800-8\omega^2 \end{vmatrix} = 0$$

$$\Rightarrow (1600-4\omega^2)(800-8\omega^2) - 800^2 = 0$$

$$\Rightarrow (4)(400-\omega^2)(8)(100-\omega^2) - 800^2 = 0$$

$$\Rightarrow (400-\omega^2)(100-\omega^2) - 20{,}000 = 0$$

$$\Rightarrow 40{,}000 - 500\omega^2 + \omega^4 - 20{,}000 = 0$$

$$\Rightarrow \omega^4 - 500\omega^2 + 20{,}000 = 0$$

$$\Rightarrow \omega^2 = \frac{500 \pm \sqrt{(500)^2 - 80{,}000}}{2} = 43.845 \ \& \ 456.155$$ (2)

$\Rightarrow \omega_{n_1} = 6.62$ rad/s ; $\omega_{n_2} = 21.36$ rad/s (Ans.)

First mode shape: use $\omega = \omega_{n_1} = 6.62$ in eq. (2):

$$\begin{bmatrix} 1424.62 & -800 \\ -800 & 449.24 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

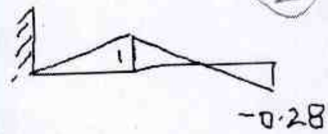
(2)

2nd mode shape

$\Rightarrow 1424.62X_1 - 800X_2 = 0 \longrightarrow (3)$

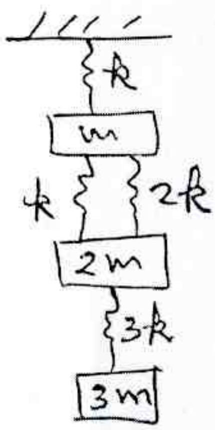
$-800X_1 + 449.24X_2 = 0 \longrightarrow (4)$

using $X_1 = 1$ in (3) $\rightarrow X_2 = 1.78$



(2)

3.)



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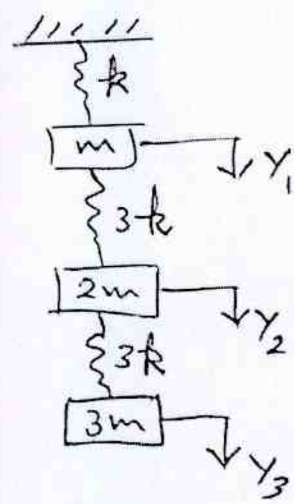


1st iteration: $Y_1 = 1$, $Y_2 = 1$, $Y_3 = 1$

$\Rightarrow F_1 = m\cdot\omega^2$; $F_2 = 2m\omega^2$; $F_3 = 3m\omega^2$ (1)

$$[a] = \begin{bmatrix} 1/k & 1/k & 1/k \\ 1/k & 4/3k & 4/3k \\ 1/k & 4/3k & 5/3k \end{bmatrix}$$

1

$$Y_1' = F_1 \cdot a_{11} + F_2 \cdot a_{12} + F_3 \, a_{13}$$

$$= \frac{m\omega^2}{k} + \frac{2m\omega^2}{k} + \frac{3m\omega^2}{k} = \frac{6m\omega^2}{k}$$

$$Y_2' = F_1 \cdot a_{21} + F_2 \cdot a_{22} + F_3 \cdot a_{23}$$

$$= \frac{m\omega^2}{k} + \frac{8m\omega^2}{3k} + \frac{12m\omega^2}{3k} = \frac{23\,m\omega^2}{3k}$$

$$Y_3' = F_1 \cdot a_{31} + F_2 \cdot a_{32} + F_3 \cdot a_{33}$$

$$= \frac{m\omega^2}{k} + \frac{8m\omega^2}{3k} + \frac{15m\omega^2}{3k} = \frac{26\,m\omega^2}{3k}$$

$\Rightarrow$ Normalizing:-

3

$$Y_1' = 1 \quad ; \quad Y_2' = 1.28 \quad ; \quad Y_3' = 1.44$$

Second iteration: $Y_1 = 1$; $Y_2 = 1.28$; $Y_3 = 1.44$

$\Rightarrow F_1 = m\omega^2$; $F_2 = 2.56\,m\omega^2$; $F_3 = 4.32\,m\omega^2$

$$\Rightarrow Y_1' = \frac{m\omega^2}{k} + \frac{2.56m\omega^2}{k} + \frac{4.32m\omega^2}{k} = \frac{7.88\,m\omega^2}{k}$$

$$Y_2' = \frac{m\omega^2}{k} + \frac{3.413\,m\omega^2}{k} + \frac{5.76m\omega^2}{k} = \frac{10.173\,m\omega^2}{k}$$

$$Y_3' = \frac{m\omega^2}{k} + \frac{3.413m\omega^2}{k} + \frac{7.2m\omega^2}{k} = \frac{11.613\,m\omega^2}{k}$$

3

$\Rightarrow$ Normalizing:-

$$Y_1' = 1 \;;\; Y_2' = 1.29 \;;\; Y_3' = 1.47$$

$$\Rightarrow 7.88\frac{m\omega^2}{k} = 1 \Rightarrow \omega_n = \sqrt{\frac{k}{7.88m}} = 0.356\sqrt{\frac{k}{m}}$$ 2

(Ans.)

4.)

$$\Rightarrow \boxed{\frac{\partial^2\theta}{\partial x^2} = \frac{1}{c^2}\cdot\frac{\partial^2\theta}{\partial t^2}} \text{ where } \boxed{c = \sqrt{\frac{G}{\rho}}}$$

c is velocity of wave propagation along the shaft.

(*) The above eq. (also called 'wave eq.') is the governing eq. of torsional vibrations.

$\Rightarrow$ We start by assuming the solution to be the product of 2 mutually exclusive functions.

$\therefore$ Let $\theta(x,t) = X(x)\cdot T(t)$

where 'X' is a function of x alone & 'T' is a function of 't' alone. 2

$$\Rightarrow \frac{\partial^2\theta}{\partial x^2} = T\cdot\frac{d^2X}{dx^2} \quad \& \quad \frac{\partial^2\theta}{\partial t^2} = X\cdot\frac{d^2T}{dt^2}$$

$\Rightarrow$ Substituting above results in wave eq.:-

$$\Rightarrow T\cdot\frac{d^2X}{dx^2} = \frac{1}{c^2}\cdot X\cdot\frac{d^2T}{dt^2} \Rightarrow \boxed{\frac{c^2}{X}\cdot\frac{d^2X}{dx^2} = \frac{1}{T}\cdot\frac{d^2T}{dt^2}}$$

we must have :- $\frac{1}{T} \cdot \frac{d^2T}{dt^2} = -a$

So that the resulting equation is that of S.H.M.

We pro-actively choose to select this unknown constant as $(-\omega_n^2)$ instead of '$-a$'.

$\therefore$ We finally put :- $\frac{c^2}{X} \cdot \frac{d^2X}{dx^2} = \frac{1}{T} \cdot \frac{d^2T}{dt^2} = -\omega_n^2$

$\Rightarrow \frac{d^2X}{dx^2} + \frac{\omega_n^2}{c^2} \cdot X = 0$ & $\frac{d^2T}{dt^2} + \omega_n^2 \cdot T = 0$

$\Rightarrow$ We already know the standard solution to these 2 differential eqns. :-

$$X = A \cdot \sin\left(\frac{\omega_n}{c}\right)x + B \cdot \cos\left(\frac{\omega_n}{c}\right)x.$$

$$\& \quad T = C \cdot \sin\omega_n t + D \cdot \cos\omega_n t.$$

where A, B, C & D are arbitrary constants which can be found by applying relevant boundary conditions.

3

Since $\theta(x,t) = X(x) \cdot T(t)$ or Simply $(X \cdot T)$

$\rightarrow$ we have :- $\underline{\theta(x,t) = \left[A \cdot \sin\frac{\omega_n x}{c} + B \cdot \cos\frac{\omega_n x}{c}\right] \times \left[C \cdot \sin\omega_n t + D \cdot \cos\omega_n t\right]}$

This is the general solution.

<u>Fixed- Fixed Condition</u> :- (i.e both ends of rod are fixed)

<u>The B.C's are</u> :- (i) $\theta\big|_{x=0} = 0$; (ii) $\theta\big|_{x=l} = 0$

1

using B.C (i):-

$$\theta = 0 = B\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

$$\Rightarrow B = 0$$

$$\therefore \quad \theta = \left[A\cdot\sin\frac{\omega_n x}{c}\right]\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

using B.C. (ii):-

$$\theta = 0 = \left[A\cdot\sin\frac{\omega_n\cdot l}{c}\right]\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

$$\Rightarrow \sin\frac{\omega_n\cdot l}{c} = 0 \quad \Rightarrow \frac{\omega_n\cdot l}{c} = n\pi \quad (n = 1, 2, \dots)$$

$$\Rightarrow \boxed{\omega_n = \frac{n c\cdot\pi}{l}}$$ This is the freq. eq.

1

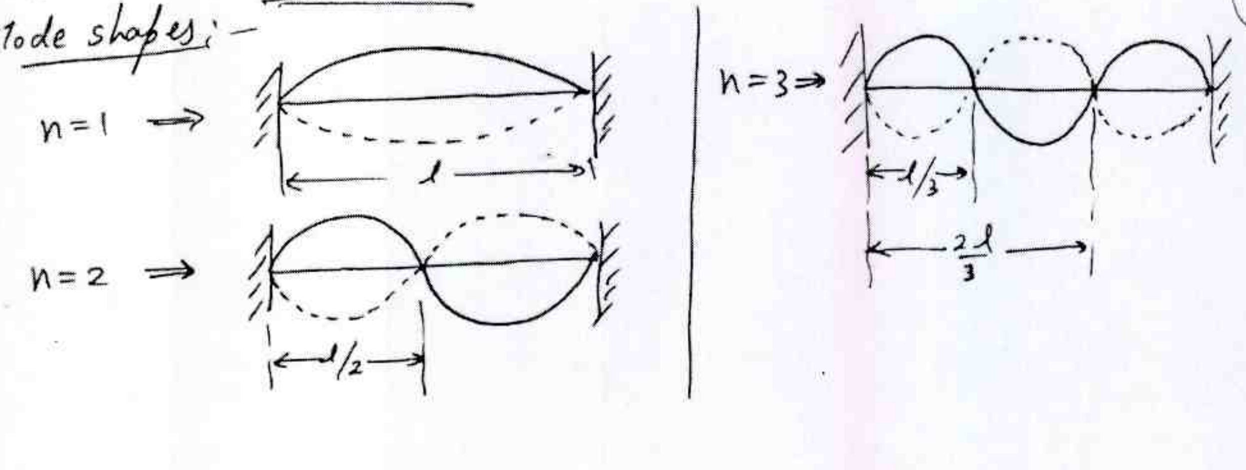
$$\therefore \quad \theta = \left[A\cdot C\cdot\sin\omega_n t + AD\cdot\cos\omega_n t\right]\cdot\sin\left(\frac{n\pi x}{l}\right)$$

Taking $\left[A\cdot C\cdot\sin\omega_n t + A\cdot D\cdot\cos\omega_n t\right] = \tau$

we get:- $\theta = \tau\cdot\sin\left(\frac{n\pi x}{l}\right)$

3

Mode shapes:-



Question Paper Solution

Branch : .........M.E............ Semester: ...VI... Subject: ....MV.............. Mid Term: ~~I~~/II/~~Extra/Imp.~~

Submitted By : Manu Augustine

5.a)

Worksite noise should be controlled to an average of below 85 dBA to protect worker hearing. Simple field fixes can significantly reduce noise exposures. Noise can be controlled by:

- Using "silenced" or muffled equipment
- Maintaining equipment and keeping tools sharp
- Locating noisy equipment as far as feasible from work areas
- Not locating noisy equipment near hard reflecting or reverberating surfaces
- Erecting barriers between the noise source and workers

5

Worksite noise is most effectively and reliably controlled at the source and in its path to the worker, not at the worker's ear.

- Muffled generators and compressors are available from equipment rental centers. Muffled compressors may produce about 75 dBA compared to 90 dBA or more for unmuffled versions.
- Doubling the distance between the noise source and the worker reduces the noise exposure about 6 dBA (3 dBA for linear sources like roads). Each additional doubling yield an additional 6 dBA of reduction. Echoing from exterior walls or interior walls and ceiling can reduce this reduction.
- Simple plywood barriers can yield a 10-12 dBA reduction in noise. The barrier should not have gaps and should be wider and higher than the line of sight between the noise source and the worker.
- Keeping cutting tools sharp and mechanical equipment well maintained reduces noise while cutting job time and operating costs.

5.b)

Total SPL due to Machines only :

$$L.P. = 10 \log \left[10^{50/10} + 10^{85/10} + 10^{92/10} + 10^{89/10} \right]$$

$$= 10 \log \left[3.777 \times 10^{9} \right]$$

2

$$= 95.77 \text{ dB}$$

Total SPL with Background Noise

$$= 10 \log \left[10^{\frac{95.77}{10}} + 10^{88/10} \right]$$

3

$$= 96.44 \text{ dB}$$

Question Paper Solution (B)

Branch : .........M.E............ Semester: ...VI... Subject: ....MV............... Mid Term: ~~I~~/II/~~Extra/Imp.~~

Submitted By : Manu Augustine

1.)

Given: $m = 10.2\ kg$; $\omega = \frac{2\pi \times 600}{60} = 62.83\ rad/s$

$Y_0 = 0.06\times10^{-3}\ m$; $k = 4\times 3200 = 12{,}800\ N/m$

$C = 400\ N\text{-}s/m$ ~~= 1600 N-s/m~~

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{12{,}800}{10.2}} = 35.42\ rad/s$$

$$r = \frac{\omega}{\omega_n} = \frac{62.83}{35.42} = 1.77$$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{400}{2\sqrt{(12{,}800\times10.2)}} = 0.55$$

(2)

(a.)

$$\frac{X_0}{Y_0} = \frac{X_0}{0.06\times10^{-3}} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \sqrt{\frac{1+(2\times0.55\times1.77)^2}{(1-1.77^2)^2+(2\times0.55\times1.77)^2}} = 0.76$$

(4)

$$\Rightarrow X_0 = 0.76\times0.06\times10^{-3} = 0.0456\times10^{-3}\ m \quad (Ans.)$$

(b.) Dynamic load $= Z_0\sqrt{(c\omega)^2+k^2}$

$$\Rightarrow \frac{Z_0}{Y_0} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{1.77^2}{\sqrt{(1-1.77^2)^2+(2\times0.55\times1.77)^2}}$$

$$= 1.08$$

(2)

$$\Rightarrow Z_0 = 1.08\times0.06\times10^{-3} = 0.0648\times10^{-3}\ m$$

$$\Rightarrow F_{dyn.} = 0.0648\times10^{-3}\sqrt{(400\times62.83)^2+(12{,}800)^2}$$

$$= 1.83\ N$$

(2)

Force per isolator $= \frac{1.83}{4} = 0.46\ N$ (Ans.)

Question Paper Solution

Branch : .........M.E............ Semester: ...VI... Subject: ....MV............... Mid Term: I/II/~~Extra/Imp.~~

Submitted By : Manu Augustine

2.) (i)

Given:- $W_1 = 400N$; $N_1 = 3000\,rpm.$

$\Rightarrow m_1 = \frac{W_1}{9.81} = 40.77\,kg.$; $\omega_1 = \omega_2 = \frac{2\pi \times N_1}{60} = 314.16\,rad/s$ (1)

$\Rightarrow \left(\frac{\omega_{n_1}}{\omega_2}\right)^2 = (0.75)^2 = \left(1+\frac{\mu}{2}\right) - \sqrt{\left(\mu + \frac{\mu^2}{4}\right)} \Rightarrow$ we get $\Rightarrow \mu = 0.34$ (2)

$\Rightarrow \left(\frac{\omega_{n_2}}{\omega_2}\right)^2 = (1.25)^2 = \left(1+\frac{\mu}{2}\right) + \sqrt{\left(\mu + \frac{\mu^2}{4}\right)} \Rightarrow$ we get $\Rightarrow \mu = 0.202$ (2)

2.) (ii)

Given: $m = 9\,kg.$; $d = 0.022\,m.$; $l = 1\,m.$; $N = 2500\,rpm.$

$e = 0.13 \times 10^{-3}\,m.$; $E = 210 \times 10^9\,N/m^2.$

$\Rightarrow \omega = \frac{2\pi N}{60} = 261.8\,rad/s$

$\Rightarrow \omega_n = \sqrt{\frac{k}{m}}$; $k = \frac{48EI}{l^3}$; $I = \frac{\pi d^4}{64} = 1.15 \times 10^{-8}\,m^4$ (2)

$\Rightarrow k = \frac{48 \times 210 \times 10^9 \times 1.15 \times 10^{-8}}{1^3} = 115920\,N/m.$

$\Rightarrow \omega_n = \sqrt{\frac{115920}{9}} = 113.49\,rad/s.$ (1)

$\therefore N_c = \frac{60\omega_n}{2\pi} = \frac{60 \times 113.49}{2\pi} = 1083.75\,rpm$ (Ans.)

$R = \frac{r^2 \cdot e}{1 - r^2}$, where $r = \frac{\omega}{\omega_n} = \frac{261.8}{113.49} = 2.31$ (2)

$\Rightarrow R = \frac{(2.31)^2 \times 0.13 \times 10^{-3}}{1 - (2.31)^2} = 1.6 \times 10^{-4}\,m$ (Ans.)

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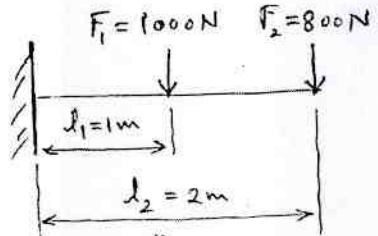
Ramnagaria, Jagatpura, Jaipur-302017

Question Paper Solution

Branch : .........M.E........... Semester: ...VI... Subject: ....MV............... Mid Term: ~~I~~/II/~~Extra/Imp.~~

Submitted By : Manu Augustine

3.)



Rayleigh's formula:

$$\boxed{\omega_n^2 = \frac{\sum_{i=1}^{N}(m_i \cdot g \cdot Y_i)}{\sum_{i=1}^{N}(m_i \cdot Y_i^2)}}$$ (2)

Given: $E = 2\times10^{11}\ N/m^2$; $I = 4\times10^{-7}\ m^4$

Here:

$$\Rightarrow \omega_n^2 = \frac{(m_1 \cdot g \cdot Y_1 + m_2 g Y_2)}{(m_1 Y_1^2 + m_2 \cdot Y_2^2)} \quad ; m_1 = \frac{1000}{9.81} = 101.94\ kg.$$

$$m_2 = \frac{800}{9.81} = 81.55\ kg.$$

$$Y_1 = F_1 \cdot a_{11} + F_2 \cdot a_{12}$$
$$Y_2 = F_1 \cdot a_{21} + F_2 \cdot a_{22}$$ (2)

$$a_{11} = \left(\frac{1 \cdot l_1^3}{3EI}\right) = 4.167\times10^{-6}\ m$$

$$a_{12} = \left(\frac{1 \cdot l_1^2(3l_2 - l_1)}{6EI}\right) = \frac{5}{6\ EI} = 1.042\times10^{-5}\ m. = a_{21}$$

$$a_{22} = \left(\frac{1 \cdot l_2^3}{3EI}\right) = \frac{8}{3EI} = 3.33\times10^{-5}\ m.$$ (3)

$$\Rightarrow Y_1 = 1000\times4.167\times10^{-6} + 800\times1.042\times10^{-5} = 0.0125\ m.$$

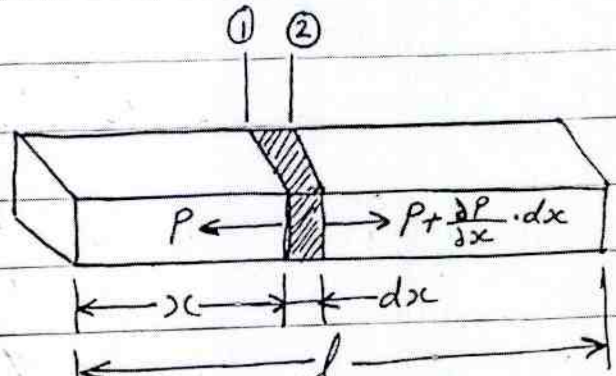
$$Y_2 = 1000\times1.042\times10^{-5} + 800\times3.33\times10^{-5} = 0.0371\ m.$$

$$\Rightarrow \omega_n^2 = \frac{(1000\times0.0125) + (800\times0.0371)}{(101.94\times0.0125^2) + (81.55\times0.0371^2)} = \frac{42.18}{0.128}$$ (3)

$$\Rightarrow \omega_n = 18.15\ rad/s\ (Ans.)$$

4.)

Consider a bar of Constant Cross-Section:-
Let Cross-Section be:- 'A'.



(1)

At any instant, let the axial forces acting on sections ① & ② be:- 'P' and '$P+\frac{\partial P}{\partial x}\cdot dx$' respectively.

→ Let displacements of sections ① & ② be:- 'u' & '$u+\frac{\partial u}{\partial x}\cdot dx$' respectively.

→ ∴ Change in length in 'dx' is :-

$$\left(u+\frac{\partial u}{\partial x}\cdot dx\right)-u=\frac{\partial u}{\partial x}\cdot dx \qquad (2)$$

→ Change in length per unit length (or strain)

$$=\frac{\left[\frac{\partial u}{\partial x}\cdot dx\right]}{dx}=\frac{\partial u}{\partial x}$$

→ By Hooke's law :- $\frac{\partial u}{\partial x} = \frac{(P/A)}{E}$

$$\Rightarrow A \cdot E \cdot \frac{\partial u}{\partial x} = P \longrightarrow (1)$$

→ Considering dynamic equilib. of slice:-

(mass) x (acc.) = Total resultant external force.

$$\Rightarrow (\rho \cdot A \cdot dx) \cdot \left(\frac{\partial^2 u}{\partial t^2}\right) = \frac{\partial P}{\partial x} \cdot dx$$

$$\Rightarrow \frac{\partial P}{\partial x} = \rho \cdot A \cdot \frac{\partial^2 u}{\partial t^2} \longrightarrow (2)$$

$$\Rightarrow \text{From (1)} \Rightarrow \frac{\partial P}{\partial x} = A \cdot E \cdot \frac{\partial^2 u}{\partial x^2} \longrightarrow (3)$$

→ From (2) & (3) :-

$$\rho \cdot \frac{\partial^2 u}{\partial t^2} = E \cdot \frac{\partial^2 u}{\partial x^2}$$

2

$$\Rightarrow \boxed{\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}} \longrightarrow (4)$$

where $c = \sqrt{\frac{E}{\rho}}$ = velocity of wave propogation in beam.

Sol. of wave eq.:- $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \cdot \frac{\partial^2 u}{\partial t^2}$

Let $u(x,t) = X(x) \cdot T(t)$

$\Rightarrow \frac{\partial^2 u}{\partial x^2} = T \cdot \frac{d^2 X}{dx^2}$; $\frac{\partial^2 u}{\partial t^2} = X \cdot \frac{d^2 T}{dt^2}$

→ substituting these in wave eq.:-

$$T \cdot \frac{d^2 X}{dx^2} = \frac{1}{c^2} \cdot X \cdot \frac{d^2 T}{dt^2} \Rightarrow \frac{c^2}{X} \cdot \frac{d^2 X}{dx^2} = \frac{1}{T} \cdot \frac{d^2 T}{dt^2}$$

$$\Rightarrow \text{Let } \frac{c^2}{X} \cdot \frac{d^2 X}{dx^2} = \frac{1}{T} \cdot \frac{d^2 T}{dt^2} = -\omega_n^2$$

$$\Rightarrow \underline{\frac{d^2 X}{dx^2} + \frac{\omega_n^2}{c^2} \cdot X = 0} \quad \& \quad \underline{\frac{d^2 T}{dt^2} + \omega_n^2 \cdot T = 0}$$

standard sol. to these differential eqns. are as follows:

$X = A \cdot \sin\left(\frac{\omega_n}{c}\right)x + B \cdot \cos\left(\frac{\omega_n}{c}\right)x$

$T = C \cdot \sin \omega_n t + D \cdot \cos \omega_n t$, where $A, B, C, \& D$ are arbitrary constants.

5

As $u = X \cdot T$, we have:- The general sol.:-

$$u(x,t) = \left[A \cdot \sin\left(\frac{\omega_n}{c}\right)x + B \cdot \cos\left(\frac{\omega_n}{c}\right)x\right]\left[C \cdot \sin \omega_n t + D \cdot \cos \omega_n t\right]$$

5.a)

The frequency *f* of an oscillating disturbance is equal to the number of times every second the disturbance passes from one extreme position to other and back to original position. The number of cycles per second is called Hertz. The frequency of a simple pure tone sound wave is called ***pitch of the tone***.

The audible frequency range: Acoustic energy associated with frequencies *above 20 kHz is inaudible* to a human being. The sound at frequencies above 20 kHz is, therefore, called ***ultrasonic sound***. The sound in the frequency range 15,000 Hz to 20,000 Hz may be audible to those people having acute (sharp) hearing. For older people, sound at frequencies above 15,000 Hz is generally not audible. Examples of common sources which emit ultrasonic sound in addition to audible sound are jet engines, high speed dental drills, spinning machines, ultrasonic cleaners and mixers. In addition to ultrasonic sounds, these sources may also emit audible sound. Ultrasonic waves have been used to detect voids, cracks and discontinuities in various structures and machine members.

Frequency range for human voice: *Human voice spreads over frequency range of 80 to 8000 Hz*. Human voice of frequency greater than 8000 Hz and smaller than 80 Hz is not common in practice. Therefore, any loss of hearing capacity (threshold shift) of an individual in the frequency range 15 to 80 Hz and 8000 to 16,000 Hz cannot be detected during human conversation.

3

5.b)

Before controlling the noise, its source must be identified and evaluated. To evaluated the noise problem, the following factors are considered :

- ☆ Type of noise
- ☆ Noise level
- ☆ Frequency distribution
- ☆ Noise sources
- ☆ Noise propagation mediums.

Control of Noise at the Source

The steps towards noise control at source are :

- ☆ To determine the causes of noise and
- ☆ What can be done to reduce the noise.

The cammon causes of noise at source are following :

(i) Mechanical shock between two machine parts.

(ii) Friction between two machine parts.

(iii) Unbalance rotating and reciprocating parts.

(iv) Vibration of large parts.

(v) Loose fittings.

(vi) Irregular fluid flow etc.

The machinery noise at sources can be controlled by following methods :

(i) **Maintenance :** The noise at source can be controlled by replacement of worn parts, balancing of unbalanced parts and proper lubrication.

(ii) **Substitution of machine part materials :** For example, the metallic gears can be replaced by Nylon gears.

(iii) **Substitution of equipment :** The substitution of the equipment which is noisy by the another equipment which is making lesser noise is another way of noise control. For example, stepped dies can be used rather than single operation dies, hydraulic process can be used rather than mechanical process, presses can be used rather than hammers and belt conveyors can be used rather than roller conveyors.

7

(iv) **Substitution of machine parts :** Machine parts can be replaced by the parts which are less noisy, for example replacing spur gears by helical gears generally reduce 10 dB of noise level.

Question Paper Solution (C)

Branch : ....ME............... Semester: ...VI... Subject: ....MV.................. Mid Term: ~~I~~/II/~~Extra/Imp.~~

Submitted By :............Manu Augustine.............

1.

$m = 600\ kg.$; $N = 200\ rpm$; $k = 200{,}000\ N/m.$; $\zeta = 0.2$

$m_o = 20\ kg.$; $e = \frac{0.2}{2} = 0.1\ m.$ (5)

$$\frac{Y_o}{\left(\frac{m_o \cdot e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta \cdot r)^2}} \; ; \; \omega_n = \sqrt{\frac{k}{m}} = 18.26\ rad/s$$

$$\omega = \frac{2\pi N}{60} = 20.94\ rad/s. \; ; \; r = \frac{\omega}{\omega_n} = 1.147.$$

(2)

$$\Rightarrow Y_o = 7.87 \times 10^{-3}\ m \text{ (Ans.)}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = -55.48^\circ \text{ or } (180^\circ + (-55.48^\circ))$$ (3)

$$\Rightarrow \phi = 124.52^\circ \text{ (Ans.)}$$

2.

Given:— $d = 0.01\ m$, $l = 1\ m$,

$m = 10\ kg.$, $e = 0.25 \times 10^{-3}\ m.$

$E = 200 \times 10^{9}\ N/m^2$; $\sigma_{Permissible} = 75 \times 10^{6}\ N/m$

$$\Rightarrow k = \frac{192\ EI}{l^3} = 18{,}854.4\ N/m$$

$$\Rightarrow \omega_n (Critical) = \sqrt{\frac{k}{m}} = 43.42\ rad/s.$$ (3)

$$\text{or } N_{critical} = 414.63\ rpm \text{ (Ans.)}$$

Question Paper Solution

Branch : ....ME............... Semester: ...VI... Subject: ....MV.................. Mid Term: ~~I~~/II/~~Extra/Imp.~~
Submitted By :...........Manu Augustine.............

$$M_{Perm.} = \frac{\sigma_{Perm.} \cdot I}{y} \; ; \; I = \frac{\pi d^4}{64} = 4.91 \times 10^{-10} \, m^4.$$

1

$$\Rightarrow M_{Perm} = 7.365 \text{ N-m.}$$

1

$$\Rightarrow F_{Perm.} = \frac{8 M_{Perm.}}{l} = 58.92 \text{ N}$$

1

$$\Rightarrow \delta_{Perm.} = \frac{F_{Perm}}{k} = 3.125 \times 10^{-3} \, m.$$

$$\Rightarrow \pm 3.125 \times 10^{-3} = \frac{e \cdot r^2}{(1-r^2)}, \text{ where } r = \left(\frac{\omega}{\omega_n}\right).$$

2

$\rightarrow$ using (+) sign:- $\quad r = 0.962 \Rightarrow \omega = \omega_1 = \underline{41.77 \text{ rad/s}}$

$\rightarrow$ using (−) sign:- $\quad r = 1.042 \Rightarrow \omega = \omega_2 = \underline{45.24 \text{ rad/s}}.$

or $N_1 = \frac{60 \times \omega_1}{2\pi} = \underline{398.87} \text{ rpm.}$

$N_2 = \frac{60 \times \omega_2}{2\pi} = \underline{432} \text{ rpm}$

2

<u>unsafe range of speed</u> :-

$N > N_1$ and $N < N_2$

<u>i.e</u> :- $\quad N_1 < N < N_2$ is unsafe.

3.

$$[I_m] = 2\times\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 3\end{bmatrix} ; [K_t] = 2\times\begin{bmatrix}5 & -5 & 0\\ -5 & 15 & -10\\ 0 & -10 & 10\end{bmatrix}$$ 2

$$[D] = [I_m]^{-1}[K_t] = \begin{bmatrix}5/2 & -5/2 & 0\\ -5 & 15 & -10\\ 0 & -10/3 & 10/3\end{bmatrix}$$

$\Rightarrow \left|[D]-\lambda[I]\right| = 0 \Rightarrow$ Simplifying the determinant, we get:-

$\lambda_1 = \omega_{n_1}^2 = 0$; $\lambda_2 = \omega_{n_2}^2 = 2.77$; $\lambda_3 = \omega_{n_3}^2 = 18.066$ 3

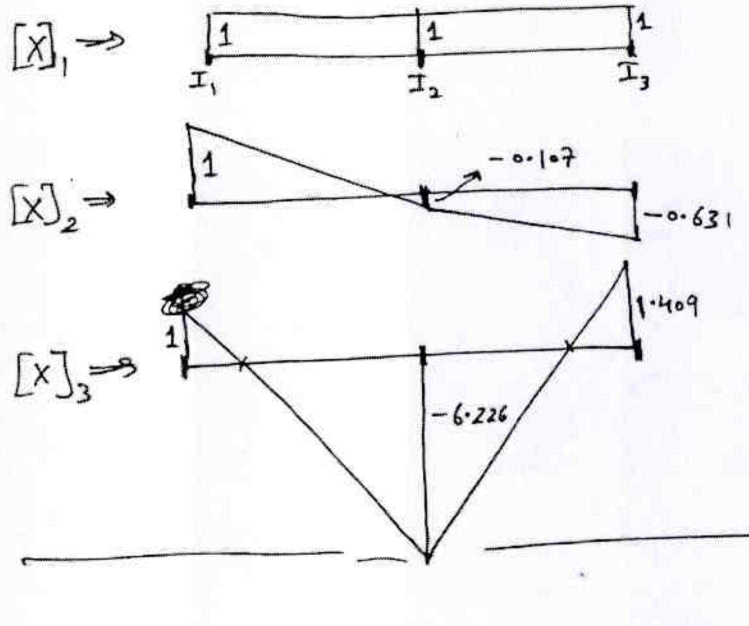
$\Rightarrow \omega_{n_1} = 0$; $\omega_{n_2} = 1.664$ rad/s ; $\omega_{n_3} = 4.25$ rad/s. (Ans.)

Using $\{[D]-\lambda[I]\}[X] = 0 \rightarrow$ we get eigen vectors for each λ :-

For $\lambda_1 \Rightarrow [X]_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$; For $\lambda_2 \Rightarrow [X]_2 = \begin{bmatrix}1\\-0.107\\-0.631\end{bmatrix}$ 3

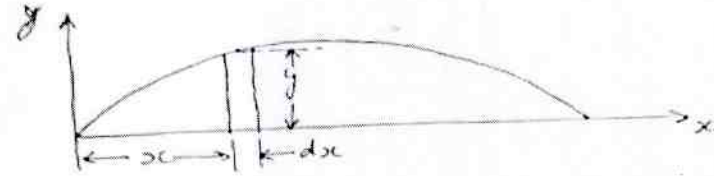
$\Rightarrow$ For $\lambda_3 \Rightarrow [X]_3 = \begin{bmatrix}1\\-6.226\\1.409\end{bmatrix}$

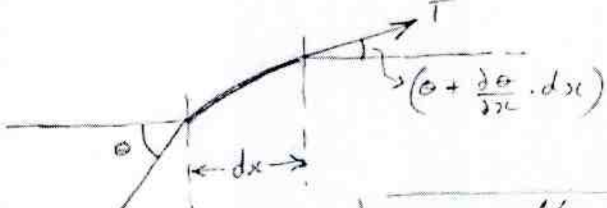
$\rightarrow$ These eigen vectors directly give us the mode shapes:-



⊛ Nodal points can be found using simple geometry from these figures. 2

4.





2

Assumptions:-
- Tension (T) is large
- Amplitude of Vibrations is small.
- T remains constant throughout length while Vibrating

1

* Since amplitude of vibrations is small, the longitudinal displacement of string elements is negligeable.
- This means there should be no resultant force in horizontal direction.

∴ Horizontal Components of T in both directions Cancel out.

→ In Vertical direction, net force is:-

$$T\left(\theta + \frac{\partial \theta}{\partial x} \cdot dx\right) - T \cdot \theta = T \cdot \frac{\partial \theta}{\partial x} \cdot dx \quad [\because \sin\theta \approx \theta]$$

If ρ = mass/length, the mass of element is $(\rho \cdot dx)$.

→ Eq. of motion:- $(\rho \cdot dx) \cdot \frac{\partial^2 y}{\partial t^2} = T \cdot \frac{\partial \theta}{\partial x} \cdot dx$

but $\theta = \frac{\partial y}{\partial x} \rightarrow \rho \cdot \frac{\partial^2 y}{\partial t^2} = T \cdot \frac{\partial^2 y}{\partial x^2} \rightarrow \boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 y}{\partial t^2}}$ 2

where $C = \sqrt{\frac{T}{\rho}}$ = velocity of wave propagation along the string.

Sol. of Wave eqn:- Tight string.

$y(x,t) = X(x)\cdot T(t)$ or $y = X\cdot T$

here X is a func. of x alone, and T is a func. of t alone.

$\Rightarrow \frac{\partial^2 y}{\partial x^2} = T\cdot\frac{d^2X}{dx^2}$ and $\frac{\partial^2 y}{\partial t^2} = X\cdot\frac{d^2T}{dt^2}$

$\Rightarrow$ Substituting in the wave eq.n

$T\cdot\frac{d^2X}{dx^2} = \frac{1}{c^2}X\cdot\frac{d^2T}{dt^2} \Rightarrow \frac{c^2}{X}\cdot\frac{d^2X}{dx^2} = \frac{1}{T}\cdot\frac{d^2T}{dt^2}$

$\Rightarrow$ Now, L.H.S is func. of x alone, and R.H.S is func. of t alone.

$\Rightarrow$ This is possible, if both sides are equal to a constant

$\Rightarrow$ this constant can be:- +ve, −ve, or 0.

$\Rightarrow$ For S.H.M $\Rightarrow$ it must be −ve.

$\therefore$ Let $\frac{c^2}{X}\cdot\frac{d^2X}{dx^2} = \frac{1}{T}\cdot\frac{d^2T}{dt^2} = -\omega^2$

$\Rightarrow \frac{d^2X}{dx^2} + \frac{\omega^2}{c^2}X = 0$ and $\frac{d^2T}{dt^2} + \omega^2 T = 0$

$\Rightarrow$ Solutions:- $X = A\cdot\sin\left(\frac{\omega}{c}\right)x + B\cdot\cos\left(\frac{\omega}{c}\right)x$ 5

$T = C\cdot\sin\omega t + D\cos\omega t.$

here A, B, C & D are arbitrary Constants which can be found by applying Bound. Conditions.

5. (i)

Sound Pressure level (SPL):

- Most commonly used decibel scale.
- Since humans do not perceive auditory pressure, but auditory intensity (which is directly proportional to p^2)

SPL is defined as:

$$L_p = 10 \log_{10}\left(\frac{p^2}{p_{ref}^2}\right), \text{ where } p = p_{rms}$$

$$\Rightarrow L_p = 20 \log_{10}\left(\frac{p}{p_{ref}}\right), \text{where } p_{ref} = 2\times10^{-5}\ N/m^2.$$

1

Sound intensity level (SIL):

$$L_i = 10 \log_{10}\left(\frac{I}{I_{ref.}}\right) \ ; \ I_{ref.} = 10^{-12}\ W/m^2$$

1

$$L_i = 10 \log_{10}\left(\frac{I}{I_{ref.}}\right) \ ; \ I = \frac{p^2}{\rho \cdot V}$$

$$\Rightarrow L_i = 10\cdot\log_{10}\left(\frac{p^2}{\rho.V\cdot I_{ref.}}\right) \ ; \left(\frac{p^2}{\rho.V.I_{ref.}}\right) = \left(\frac{p^2}{p_{ref.}^2}\right)\cdot\left(\frac{p_{ref.}^2}{\rho.V.I_{ref.}}\right)$$

Now, $p_{ref.} = 2\times10^{-5}\ N/m^2$; $I_{ref.} = 10^{-12}\ W/m^2$; $(\rho.V) = 415\ kg/m^2\text{-}s$

3

$$\Rightarrow L_i = 10\log_{10}\left(\frac{p^2}{p_{ref.}^2}\right) + 10\log_{10}\left(\frac{p_{ref}^2}{\rho.V.I_{ref}}\right)$$

$$\Rightarrow \boxed{L_i = L_p - 0.16} \text{ or } \boxed{L_i \approx L_p}$$

Question Paper Solution

5. (ii)

SPL due to Component 1 alone :-

$$L_{P_1} = 10 \log_{10}\left[10^{90/10} - 10^{75/10}\right] = 89.86 \text{ dB} \quad 1$$

Similarly for other components:-

$$L_{P_2} = 10 \log_{10}\left[10^{95/10} - 10^{75/10}\right] = 94.96 \text{ dB} \quad 1$$

$$L_{P_3} = 10 \log_{10}\left[10^{85/10} - 10^{75/10}\right] = 84.54 \text{ dB} \quad 1$$

$$L_{P_4} = 10 \log_{10}\left[10^{80/10} - 10^{75/10}\right] = 78.35 \text{ dB.} \quad 1$$

∴ Total SPL due to overall machine with No background noise is :-

$$L_P = 10 \log\left[10^{89.86/10} + 10^{94.96/10} + 10^{84.54/10} + 10^{78.35/10}\right] \quad 1$$

$$= \underline{96.49 \text{ dB}} \quad (\text{Ans.})$$

Question Paper Solution (D)

Branch : ....ME............... Semester: ...VI... Subject: ....MV.................. Mid Term: I/II/~~Extra/Imp.~~

Submitted By :............Manu Augustine.............

1.

Given:- $m = 900\,kg$; $k = 300{,}000\,N/m$

$\zeta = 0.5$; $V = 90 \times \frac{5}{18} = 25\,m/s$; $\lambda = 5\,m$; $Y_0 = 0.5\,m$

5

$$\frac{X_0}{Y_0} = \frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} \; ; \; r = \frac{\omega}{\omega_n} \; ; \; \omega_n = \sqrt{\frac{k}{m}} = 18.26\,rad/s$$

$\lambda = V \times T \Rightarrow T = \frac{\lambda}{V} = \frac{5}{25} = 0.2\,sec. \rightarrow f = \frac{1}{T} = 5\,Hz.$ 2

$\omega = 2\pi f = 31.42\,rad/s.$

$\Rightarrow r = \frac{\omega}{\omega_n} = \frac{31.42}{18.26} = 1.72$

$\Rightarrow \frac{X_0}{Y_0} = 0.76$; $Y_0 = 0.5\,m \Rightarrow X_0 = \underline{\underline{0.38\,m}}$ (Ans.) 3

2.

Eq. of motion for disc-1:-

$$I_1 \cdot \ddot{\theta}_1 + K_{t_1} \theta_1 + K_{t_2}(\theta_1 - \theta_2) = 0$$

$$\Rightarrow I_1 \cdot \ddot{\theta}_1 + K_{t_1}\theta_1 + 2K_{t_1}\theta_1 - 2K_{t_1}\theta_2 = 0 \quad (\because K_{t_2} = 2K_{t_1})$$

$$\Rightarrow I_1 \cdot \ddot{\theta}_1 + 3K_{t_1}\theta_1 - 2K_{t_1}\theta_2 = 0$$

1

Eq. of motion for disc-2:-

$$I_2 \ddot{\theta}_2 + K_{t_2}(\theta_2 - \theta_1) = 0$$

$$\Rightarrow 2I_1\ddot{\theta}_2 + 2K_{t_1}\theta_2 - 2K_{t_1}\theta_1 = 0 \quad (\because I_2 = 2I_1 \;\&\; K_{t_2} = 2K_{t_1})$$

Let the sol. be:- $\left.\begin{array}{l}\theta_1 = \phi_1 \cdot \sin\omega t \\ \theta_2 = \phi_2 \cdot \sin\omega t\end{array}\right\} \Rightarrow \begin{array}{l}\ddot{\theta}_1 = -\omega^2 \phi_1 \cdot \sin\omega t \\ \ddot{\theta}_2 = -\omega^2 \phi_2 \cdot \sin\omega t\end{array}$ 1

Here ϕ_1 & ϕ_2 are amplitudes of θ_1 & θ_2 respectively.

⇒ using these in eq. of 1st disc:-

$$I_1(-\omega^2\phi_1 \cdot \sin\omega t) + 3K_{t_1}(\phi_1 \cdot \sin\omega t) - 2K_{t_1}(\phi_2 \cdot \sin\omega t) = 0$$

$$\Rightarrow -\omega^2 I_1\phi_1 + 3K_{t_1}\cdot\phi_1 - 2K_{t_1}\cdot\phi_2 = 0$$

$$\Rightarrow \left(\frac{\phi_2}{\phi_1}\right) = \frac{(-\omega^2 I_1 + 3K_{t_1})}{2K_{t_1}} \longrightarrow (1)$$

⇒ Similarly from eq. of 2nd disc:-

$$2I_1(-\omega^2\phi_2) + 2K_{t_1}\phi_2 - 2K_{t_1}\cdot\phi_1 = 0$$

$$\Rightarrow \left(\frac{\phi_2}{\phi_1}\right) = \frac{2K_{t_1}}{(-2\omega^2 I_1 + 2K_{t_1})} \longrightarrow (2)$$

⇒ Equating (1) & (2) :-

$$\left(\frac{\phi_2}{\phi_1}\right) = \frac{(-\omega^2 I_1 + 3K_{t_1})}{2K_{t_1}} = \frac{2K_{t_1}}{(-2\omega^2 I_1 + 2K_{t_1})}$$

⇒ Re-arranging:- we get :-

$$2\omega^4 I_1^2 - 6\omega^2 I_1 K_{t_1} - 2\omega^2 I_1 K_{t_1} + 6K_{t_1}^2 = 4K_{t_1}^2$$

$$\Rightarrow 2\omega^4 I_1^2 - 8\omega^2 I_1 \cdot K_{t_1} + 2K_{t_1}^2 = 0$$

$$\Rightarrow (I_1^2)\cdot\omega^4 - (4I_1\cdot K_{t_1})\omega^2 + K_{t_1}^2 = 0$$

⇒ This is a quadratic eq. in (ω^2). It's roots are:-

$$(\omega^2) = \frac{(4I_1\cdot K_{t_1}) \pm \sqrt{16I_1^2\cdot K_{t_1}^2 - 4I_1^2\cdot K_{t_1}^2}}{2I_1^2}$$

2

$$\Rightarrow (\omega^2) = 2\left(\frac{K_{t_1}}{I_1}\right) \pm \sqrt{\frac{12I_1^2\cdot K_{t_1}^2}{4I_1^4}} = 2\left(\frac{K_{t_1}}{I_1}\right) \pm \sqrt{3}\left(\frac{K_{t_1}}{I_1}\right)$$

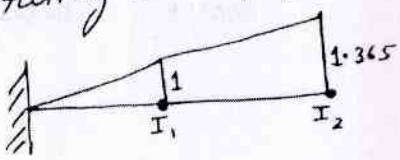
$\cancel{\Rightarrow \omega_1 = 1.932\sqrt{\frac{K_{t_1}}{I_1}}\ ;\ \omega_2 =}$ $\Rightarrow \omega_1 = 0.52\sqrt{\frac{K_{t_1}}{I_1}}\ ;\ \omega_2 = 1.93\sqrt{\frac{K_t}{I_1}}$

Using given values $\left.\begin{array}{l} K_{t_1} = 20\ \text{N-m/rad.} \\ \& I_1 = 10\ \text{kg-m}^2 \end{array}\right\} \Rightarrow \omega_1 = 0.735\ \text{rad/s}\ ;\ \omega_2 = 2.73\ \text{rad/s}$ 2

Normal modes & mode shapes :-

using eq. ① :- and substituting $\omega = \omega_1$:-

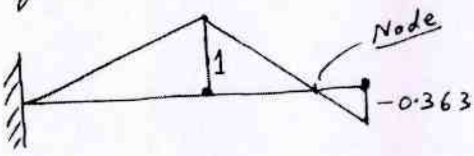
$$\left(\frac{\phi_2}{\phi_1}\right) = 1.365 \Rightarrow \quad (I^{st}\ \text{mode})$$



(2)

using eq. ① :- and substituting $\omega = \omega_2$:-

$$\left(\frac{\phi_2}{\phi_1}\right) = -0.363 \Rightarrow$$



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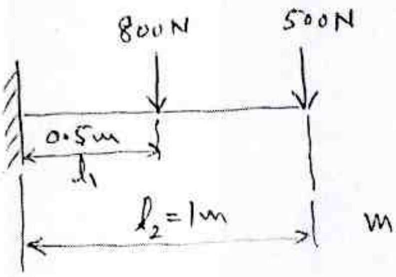
3.



Given: $E = 2\times10^{11}\ N/m^2$

$I = 4\times10^{-7}\ m^4$.

$m_1 = 81.55\ kg.$; $m_2 = 50.97\ kg.$

$$a_{11} = \frac{1 \cdot l_1^3}{3EI} = 5.21\times10^{-7}\ m. \ ; \quad a_{12} = a_{21} = \frac{1\cdot l_1^2(3l_2 - l_1)}{6EI} = \frac{0.625}{6EI} = 1.3\times10^{-6}\ m.$$

(3)

$$a_{22} = \frac{1\cdot l_2^3}{3EI} = 4.17\times10^{-6}\ m.$$

First Iteration: $Y_1 = 1$; $Y_2 = 1 \Rightarrow F_1 = 81.55\omega^2$; $F_2 = 50.97\omega^2$

$$\Rightarrow Y_1' = F_1 \cdot a_{11} + F_2 \cdot a_{12} = 1.09\times10^{-4}\ \omega^2.$$

$$Y_2' = F_1 \cdot a_{21} + F_2 \cdot a_{22} = 3.18\times10^{-4}\ \omega^2.$$

(3)

$\Rightarrow$ Normalizing: $Y_1' = 1$; $Y_2' = 2.92$

Second Iteration: $Y_1 = 1$; $Y_2 = 2.92$

$\Rightarrow F_1 = 81.55\omega^2$; $F_2 = 148.83\omega^2$

$\Rightarrow Y_1' = 2.36\times10^{-4}\omega^2$ 3

$Y_2' = 7.27\times10^{-4}\omega^2$

$\Rightarrow$ Normalizing: $Y_1' = 1$; $Y_2' = 3.08$

$\Rightarrow 2.36\times10^{-4}\omega^2 = 1 \Rightarrow \omega^2 = 4237.29 \text{ rad}^2/s^2$ 1

$\therefore$ Fundamental $\omega_n = \sqrt{4237.29} = 65.1$ rad/s (Ans.)

4.

Wave equation: $$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}}$$ 2

Sol. of Wave eqn.:- Tight string.

$y(x,t) = X(x)\cdot T(t)$ or $y = X\cdot T$

here X is a func. of x alone, and T is a func. of t alone.

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = T\cdot\frac{d^2X}{dx^2} \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = X\cdot\frac{d^2T}{dt^2}$$

$\Rightarrow$ Substituting in the wave eq.n

$$T\cdot\frac{d^2X}{dx^2} = \frac{1}{c^2}X\cdot\frac{d^2T}{dt^2} \Rightarrow \frac{c^2}{X}\cdot\frac{d^2X}{dx^2} = \frac{1}{T}\cdot\frac{d^2T}{dt^2}$$

→ Now, L.H.S is func. of x alone, and R.H.S is func. of t alone.
→ This is possible, if both sides are equal to a constant
⟹ this constant can be :- +ve, −ve, or 0.
→ For S.H.M ⟹ it must be −ve.

$$\therefore \text{ Let } \frac{c^2}{X}\cdot\frac{d^2X}{dx^2} = \frac{1}{T}\cdot\frac{d^2T}{dt^2} = -\omega^2$$

$$\Rightarrow \frac{d^2X}{dx^2} + \frac{\omega^2}{c^2}X = 0 \quad \text{and} \quad \frac{d^2T}{dt^2} + \omega^2 T = 0$$

4

→ Solutions:- $X = A\cdot\sin\left(\frac{\omega}{c}\right)x + B\cdot\cos\left(\frac{\omega}{c}\right)x$

$T = C\cdot\sin\omega t + D\cos\omega t.$

Fixed- Fixed Condition :- (i.e both ends are fixed)

The B.C's are :- (i) $y\big|_{x=0} = 0$; (ii) $y\big|_{x=l} = 0$

using B.C (i):-

$$y = 0 = B\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

$$\Rightarrow B = 0$$

$$\therefore \; y = \left[A\cdot\sin\frac{\omega_n x}{c}\right]\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

using B.C. (ii):-

$$y = 0 = \left[A\cdot\sin\frac{\omega_n\cdot l}{c}\right]\left[C\cdot\sin\omega_n t + D\cdot\cos\omega_n t\right]$$

Question Paper Solution

Branch : ....ME............... Semester: ...VI... Subject: ....MV.................. Mid Term: I/II/~~Extra/Imp.~~
Submitted By :............Manu Augustine.............

$$\Rightarrow \sin\frac{\omega_n \cdot l}{c} = 0 \Rightarrow \frac{\omega_n \cdot l}{c} = n\pi \quad (n = 1, 2, \ldots)$$

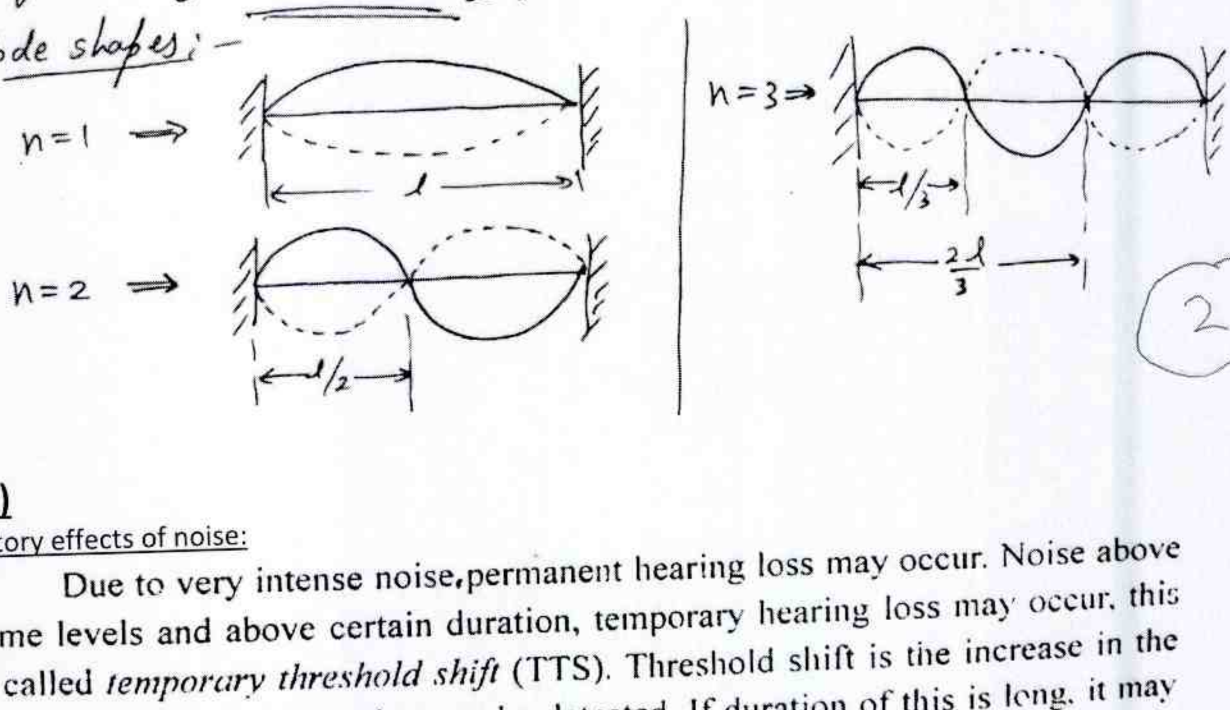
$$\Rightarrow \boxed{\omega_n = \frac{n c \cdot \pi}{l}}$$ This is the freq. eq. 2

$$\therefore y = [A \cdot C \cdot \sin\omega_n t + AD \cdot \cos\omega_n t] \cdot \sin\left(\frac{n\pi x}{l}\right)$$

Taking $[A \cdot C \cdot \sin\omega_n t + A \cdot D \cdot \cos\omega_n t] = \tau$

we get:- $y = \tau \cdot \sin\left(\frac{n\pi x}{l}\right)$

Mode shapes:-



2

5. (i)

Auditory effects of noise:

Due to very intense noise, permanent hearing loss may occur. Noise above some levels and above certain duration, temporary hearing loss may occur, this is called *temporary threshold shift* (TTS). Threshold shift is the increase in the minimum sound pressure that can be detected. If duration of this is long, it may cause permanent threshold shift (PTS). Besides threshold, noise exposure over a long period may causes distortion of clarify and quality of auditory sensation. 3

Noise can permanently damage the inner ear cells. Noise having frequencies between 2 kHz and 6 kHz produces relatively higher TTS. TTS and PTS can act additively with natural hearing loss with age.

At 70 years age loss may be 30 dB for 4 kHz frequency tones. Higher frequencies produce more TTS. TTS occurs at frequencies half to one octave higher than these of the noises causing it. Frequencies around 4 kHz are must effected by noise and are the earliest to be affected.

5. (ii)

Non-auditory effects of noise:

(i) It causes interference with speech communication.

(ii) Noise disturbs sleep, though the mechanism to disturbance of sleep is complicated.

(iii) Noise can cause irritation or mentally disturbance.

(iv) Noise can interfere with performance of complicated tasks in which speech communication is required.

3

(v) Workers exposed high noise levels have been found to have higher incidence of ENT problems and cardiovascular disorders.

(vi) Noise may reduce the privacy of human beings.

5. (iii)

Major sources of noise:

Type of source
Transportation
Light industries, cottage industries
Entertainment
Retail trade
Heavy industry
A.C. and ventilating units in offices and public buildings
Construction activities
Social services (festivals, temples, processions, Public meetings, etc.)
Aircrafts (civil and military both)

Source/activity that generates noise
Loudspeakers
Traffic
Crackers
Festivals
Marriage functions
Open air cinema shows
Industries
Air traffic
Radio/television
Barking of dogs
Hawkers

Machine/industrial process
Steel plate riveting
Oxygen torch
Pneumatic metal chipper
Boilermaker shop
Textile loom
Circular saw
Pile driver at 15 m
Farm tractor/powered lawn mower
Newspaper press
Coal face drill
Bench lathe
Milling machine
Bed press
High speed drill
Key press machine

4